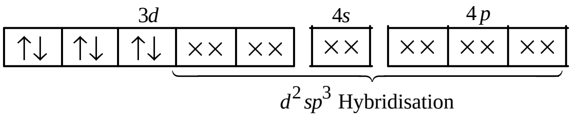
# SOLUTIONS



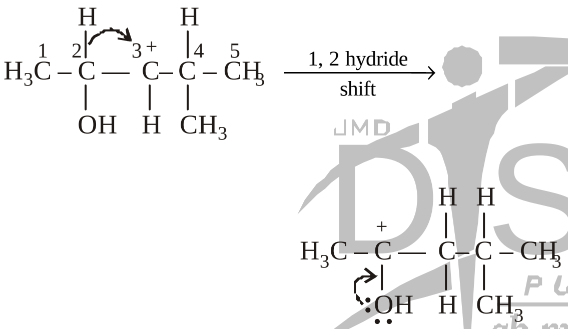
## CHEMISTRY

**1. (a)** Chromium in $Cr(CO)_6$ is in zero oxidation state and has $[Ar]^{18}\, 3d^5 4s^1$ as the electronic configuration. However, CO is a strong ligand, hence pairing up of electrons takes place leading to following configuration in $Cr(CO)_6$.




Since the complex has no unpaired electron, its magnetic moment is zero.

**2. (d)** Migrating tendency of hydride is greater than that of alkyl group. Further migration of hydride from C–2 gives more stable carbocation (stabilized by +R effect of OH group and +I and hyperconjugative effects of methyl group).




More stable carbocation

**3. (d)** $\log k = \log A - \dfrac{E_a}{2.303RT}$ …(1)

Also given $\log k = 6.0 - (2000)\dfrac{1}{T}$ …(2)

On comparing equations, (1) and (2)

$\log A = 6.0 \Rightarrow A = 10^6\ s^{-1}$

and $\dfrac{E_a}{2.303\ R} = 2000$ ;

$\Rightarrow E_a = 2000 \times 2.303 \times 8.314 = 38.29\ kJ\ mol^{-1}$

**4. (b)** $H_2C = N^+ = N^-$ > $H_2C^- - N^+ \equiv N$

I: Octet complete, 6 covalent bond, –ve charge on N

III: Octet complete, 6 covalent bond, –ve charge on C

$> H_2C^+ - N = N^- > H_2C^- - N = N^+$

II: Octet incomplete, 5 covalent bond, –ve charge on N

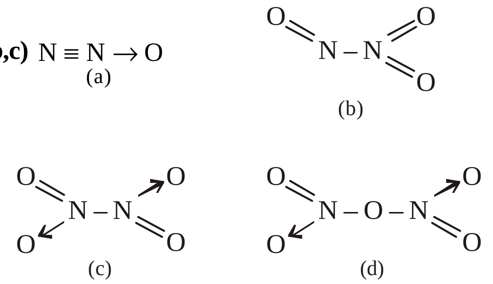
IV: Octet incomplete, 5 covalent bond, –ve charge on C

**5. (a,b,c)** Lower amines like $NH_3$, $CH_3NH_2$ and $(CH_3)_2NH$ break diborane molecule unsymmetrically, while larger amines like $(CH_3)_3N$, $C_5H_5N$ break diborane in symmetrical manner.

$$B_2H_6 + 2NH_3 \longrightarrow [H_2B(NH_3)_2]^+[BH_4]^-$$

$$B_2H_6 + 2NH_3 \longrightarrow 2H_3B \longleftarrow N(CH_3)_3$$

**6. (a,b,c)** $N \equiv N \to O$ (a)




**7. (a,b,d)** The species having less reduction potential with respect to $NO_3^-$ (E° = + 0.96 V) will be oxidised by $NO_3^-$. These species are V, Fe and Hg.

**8. (b,c)** We know that carbohydrates having acetal linkage are non-reducing while that with hemiacetal linkage are reducing. In the give structure,

**X** has acetal linkage, hence non-reducing.

**Y** has hemiacetal linkage, hence reducing.

Further **X** is α-anomer, while **Y** is β-anomer of D-(+)-glucose.

**9. (a,d)** Internal energy and molar enthalpy are state functions. Work (reversible or irreversible) is a path function.

**10. (A) –p, s ; (B) –q, s ; (C) –r, t ; (D) –q, t**

(A) $3Cu + 8\ HNO_3$ (dil.)

$$\longrightarrow \underset{p}{2NO} + \underset{s}{Cu(NO_3)_2} + 4H_2O$$

(B) $Cu + 4HNO_3$ (conc.)

$$\longrightarrow \underset{q}{2NO_2} + \underset{s}{Cu(NO_3)_2} + 2H_2O$$

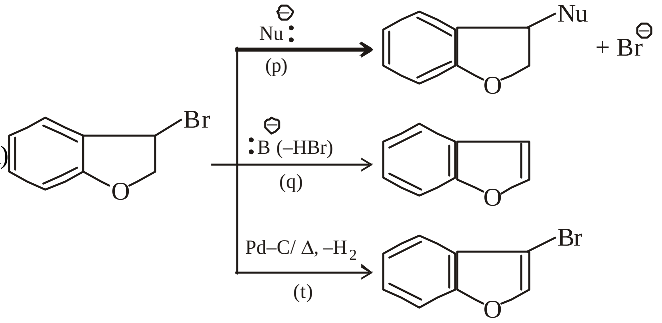
(C) $4Zn + 10HNO_3$(dil.)

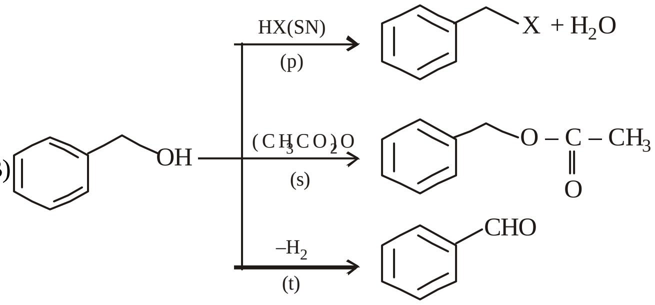
$$\longrightarrow \underset{t}{4Zn(NO_3)_2} + \underset{r}{N_2O} + 5H_2O$$

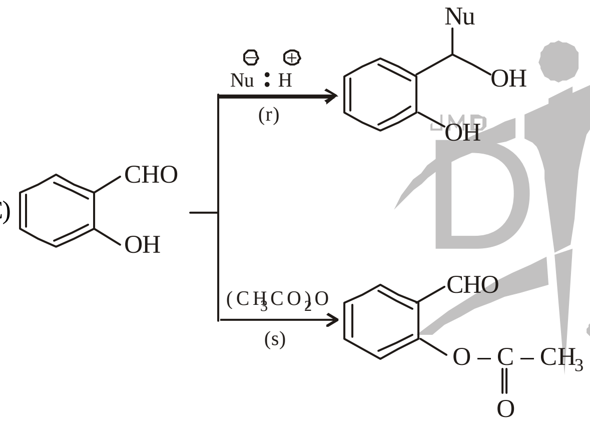
(D) $Zn + 4HNO_3$ (conc.)

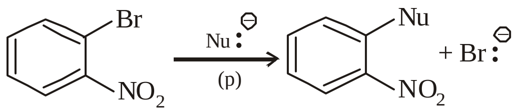
$$\longrightarrow \underset{t}{Zn(NO_3)_2} + \underset{q}{2NO_2} + 2H_2O$$

**11. (A) – p, q, t ; (B) – p, s, t ; (C) – r, s ; (D) – p**

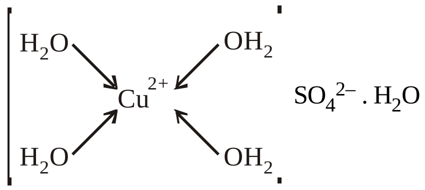
(A)


(B)


(C)


(D)


**12.** The number of water molecules directly bonded to the metal centre in $CuSO_4.5H_2O$ is 4.



**13.** pH of sodium salt of weak acid

$$= \frac{1}{2}(pK_w + pK_a + \log C)$$

$$= \frac{1}{2}(14 + 4 - 2) = 8$$

**14.** $2MnO_2 + 4KOH + O_2 \longrightarrow 2K_2MnO_4 + 2H_2O$

Oxidation number of Mn in $K_2MnO_4$ is 6

$K_2MnO_4$ ; $2 + x - 8 = 0$

$x = 6$

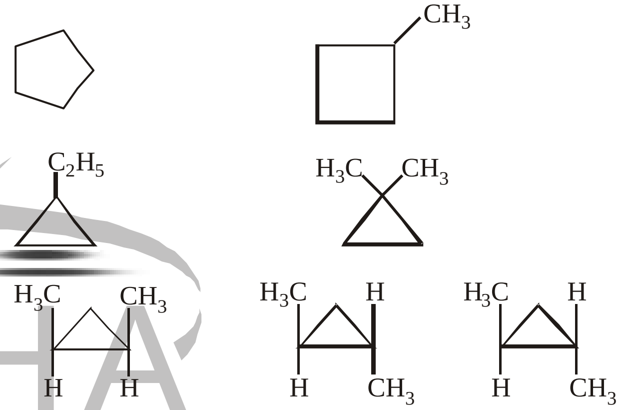
**15.** $v_{rms}$ of X $= \sqrt{\frac{3RT_x}{M_x}}$ ; $v_{mp}$ of Y $= \sqrt{\frac{2RT_y}{M_y}}$

Given $v_{rms} = v_{mp}$

$$\Rightarrow \sqrt{\frac{3RT_x}{M_x}} = \sqrt{\frac{2RT_y}{M_y}}$$

$$\Rightarrow M_y = \frac{2RT_yM_x}{3RT_x} = \frac{2\times60\times40}{3\times400} = 4$$

**16.** The seven possible cyclic structural and stereoisomers are



*cis*, (*meso*) *trans*, enantiomers

**17.** Coordination number of Al is 6. It exists in *ccp* lattice with 6 coordinate layer structure.

**18.** Energy released by combustion of 3.5 g gas

$= 2.5 \times (298.45 - 298)$ kJ

Energy released by 1 mole of gas $= \frac{2.5\times0.45}{3.5/28} = 9 kJ mol^{-1}$

**19.** $_{92}U^{238} \xrightarrow{-6\alpha} {}_{80}X^{214} \xrightarrow{-2\beta} {}_{82}Pb^{214}$

Hence total number of particles emitted are 2 +6 = 8

## MATHEMATICS

**20. (c)** The line has +ve and equal direction cosines, these are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ or direction ratios are 1, 1, 1. Also the lines passes through *P* (2, – 1, 2).

∴ Equation of line is

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-2}{1} = \lambda \text{ (say)}$$

Let $Q(\lambda+2, \lambda-1, \lambda+2)$ be a point on this line where it meets the plane

$$2x + y + z = 9$$

Then *Q* must satisfy the $eq^n$ of plane i.e. $2(\lambda+2)+\lambda-1+\lambda+2=9$

$\Rightarrow \quad \lambda = 1$

$\therefore$ $Q$ has coordintes (3, 0, 3)

Hence the length of line segments $PQ$

$= \sqrt{(2-3)^2+(-1-0)^2+(2-3)^2} = \sqrt{3}$

**21.** **(c)** The given ellipse is $\dfrac{x^2}{4^2}+\dfrac{y^2}{2^2}=1$

such that $a^2 = 16$ and $b^2 = 4$

$\therefore \quad e^2 = 1-\dfrac{4}{16}=\dfrac{3}{4} \Rightarrow e=\dfrac{\sqrt{3}}{2}$

Let $P(4\cos\theta, 2\sin\theta)$ be any point on the ellipse, then equation of normal at $P$ is

$4x\sin\theta - 2y\cos\theta = 12\sin\theta\cos\theta$

$\Rightarrow \quad \dfrac{x}{3\cos\theta}-\dfrac{y}{6\sin\theta}=1$

$\therefore$ $Q$, the point where normal at $P$ meets $x$–axis, has coordinates $(3\cos\theta, 0)$

$\therefore$ Mid point of $PQ$ is $M\left(\dfrac{7\cos\theta}{2}, \sin\theta\right)$

For locus of point $M$ we consider

$x=\dfrac{7\cos\theta}{2}$ and $y=\sin\theta$

$\Rightarrow \quad \cos\theta=\dfrac{2x}{7}$ and $\sin\theta = y$

$\Rightarrow \quad \dfrac{4x^2}{49}+y^2=1 \qquad ...(1)$

Also the latus rectum of given ellipse is

$x=\pm ae = \pm 4\times\dfrac{\sqrt{3}}{2}=\pm 2\sqrt{3}$ or $x=\pm 2\sqrt{3} \quad ...(2)$

Solving equations (1) and (2), we get

$\dfrac{4\times 12}{49}+y^2=1 \Rightarrow y^2=\dfrac{1}{49}$ or $y=\pm\dfrac{1}{7}$

$\therefore$ The required points are $\left(\pm 2\sqrt{3}, \pm\dfrac{1}{7}\right)$.

**22.** **(c)** Given that for an A.P, $S_n = cn^2$

Then $T_n = S_n - S_{n-1} = cn^2 - c(n-1)^2$

$= (2n-1)c$

$\therefore$ Sum of squares of $n$ terms of this A.P

$= \sum T_n^2 = \sum (2n-1)^2 c^2$

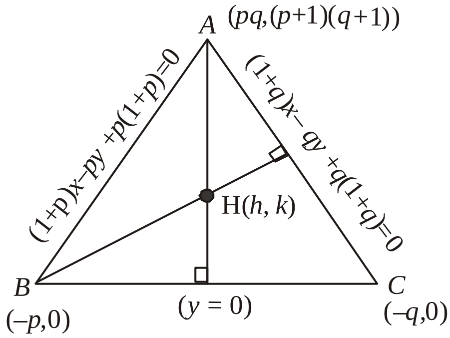
$= c^2\left[4\sum n^2 - 4\sum n + n\right]$

$= c^2\left[\dfrac{4n(n+1)(2n+1)}{6}-\dfrac{4n(n+1)}{2}+n\right]$

$= c^2 n\left[\dfrac{2(2n^2+3n+1)-6(n+1)+3}{3}\right]$

$= c^2 n\left[\dfrac{4n^2-1}{3}\right]=\dfrac{n(4n^2-1)c^2}{3}$

**23.** **(d)** The triangle is formed by the lines




$AB: (1+p)x - py + p(1+p) = 0$

$AC: (1+q)x - qy + q(1+q) = 0$

$BC: y = 0$

So that the vertices are

$A(pq, (p+1)(q+1)),\ B(-p, 0), C(-q, 0)$

Let $H(h,k)$ be the orthocentre of $\Delta ABC$. Then as $AH \perp BC$ and passes through $A(pq, (p+1)(q+1))$

The eq$^n$ of $AH$ is $x = pq$

$\therefore \quad h = pq \qquad ...(1)$

Also $BH$ is perpendicular to $AC$

$\therefore \quad m_1 m_2 = -1 \Rightarrow \dfrac{k-0}{h+p}\times\dfrac{1+q}{q}=-1$

$\Rightarrow \quad \dfrac{k}{pq+p}\times\dfrac{1+q}{q}=-1 \qquad$ (using eq$^n$ (1)

$\Rightarrow \quad k = -pq \qquad ...(2)$

From (1) and (2) we observe $h + k = 0$

$\therefore$ Locus of $(h, k)$ is $x + y = 0$ which is a straight line.

**24.** **(b, c, d)** We have, $f(x) = x\cos\dfrac{1}{x},\ x \ge 1$

$\therefore \quad f'(x)=\cos\dfrac{1}{x}+\dfrac{1}{x}\sin\dfrac{1}{x}$

$\lim_{x\to\infty} f'(x) = \cos 0 + (0) \times$ (some finite value)

$\Rightarrow \quad \lim_{x\to\infty} f'(x) = 1$

Also $f''(x)=\dfrac{1}{x^2}\sin\dfrac{1}{x}-\dfrac{1}{x^2}\sin\dfrac{1}{x}-\dfrac{1}{x^3}\cos\dfrac{1}{x}$

$\Rightarrow \quad f''(x)=\dfrac{-1}{x^3}\cos\dfrac{1}{x}<0,\ \forall x \in [1, \infty)$

$\Rightarrow \quad f'(x)$ is strictly decreasing in $[1, \infty)$

$\therefore \quad f'(x) > \lim_{x\to\infty} f'(x)$

$\Rightarrow \quad \dfrac{f(x+2)-f(x)}{(x+2)-x}>1$

$\Rightarrow \quad f(x+2)-f(x)>2$

**25.** **(a, d)** Let $P(at^2, 2at)$ be any point on the parabola

$$y^2 = 4ax.$$

Then tangent to parabola at $P$ is $y = \frac{x}{t} + at$ which meets the axis of parabola *i.e* $x$-axis at $T(-at^2, 0)$.

Also normal to parabola at $P$ is $tx + y = 2at + at^3$ which meets the axis of parabola at $N(2a + at^2, 0)$

Let $G(x, y)$ be the centriod of $\Delta PTN$, then

$$x = \frac{at^2 - at^2 + 2a + at^2}{3} \text{ and } y = \frac{2at}{3}$$

$$\Rightarrow \quad x = \frac{2a + at^2}{3} \text{ and } y = \frac{2at}{3}$$

Eliminating $t$ from above, we get the locus of centriod $G$ as

$$3x = 2a + a\left(\frac{3y}{2a}\right)^2 \Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2}{3}a\right)$$

which is a parabola with vertex $\left(\frac{2a}{3}, 0\right)$, directrix as $x - \frac{2a}{3} = -\frac{a}{3}$ or $x = \frac{a}{3}$, latus rectum as $\frac{4a}{3}$ and focus as $(a, 0)$.

**26.** **(a, b, c)** We have

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx \quad ....(1)$$

$$\Rightarrow \quad I_n = \int_{-\pi}^{\pi} \frac{\sin n(-x)}{(1+\pi^{-x})\sin(-x)} dx$$

$$\left[\text{Using } \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$$\Rightarrow \quad I_n = \int_{-\pi}^{\pi} \frac{\pi^x \sin nx}{(1+\pi^x)\sin x} dx \quad ...(2)$$

Adding equation (1) and (2), we get

$$2I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{\sin x} dx = 2\int_0^{\pi} \frac{\sin nx}{\sin x} dx$$

[as integrand is an even function]

$$\Rightarrow \quad I_n = \int_0^{\pi} \frac{\sin nx}{\sin x} dx$$

Now $I_{n+2} - I_n = \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x} dx$

$$= \int_0^{\pi} \frac{2\cos(n+1)x \sin x}{\sin x} dx = 2\int_0^{\pi} \cos(n+1)x\,dx$$

$$= 2\left[\frac{\sin(n+1)x}{n+1}\right]_0^{\pi} = 0$$

$\therefore \quad I_{n+2} = I_n$

Also $I_1 = \int_0^{\pi} 1dx = \pi$ and $I_0 = 0$

Hence $\sum_{m=1}^{10} I_{2m+1} = I_3 + I_5 + I_7 + ...I_{21}$

$= 10\,I$ (using $I_{n+2} = I_n$)

$= 10\ \pi$

and $\sum_{m=1}^{10} I_{2m} = I_2 + I_4 + I_6 + \ ... + I_{20}$

$= 20 \times I_0$ (using $I_{n+2} = I_n$)

$= 20 \times 0 = 0$

**27.** **(c, d)** We have

$$\sum_{m=1}^{6} \text{cosec}\left[\theta + \frac{(m-1)\pi}{4}\right]\text{cosec}\left[\theta + \frac{m\pi}{4}\right] = 4\sqrt{2}$$

$$\Rightarrow \sum_{m=1}^{6} \frac{\sin\frac{\pi}{4}}{\sin\left[\theta + \frac{(m-1)\pi}{4}\right]\sin\left[\theta + \frac{m\pi}{4}\right]} = 4$$

$$\Rightarrow \sum_{m=1}^{6} \frac{\sin\left[\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right]}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4$$

$$\Rightarrow \sum_{m=1}^{6} \frac{\left[\begin{array}{l}\sin\left(\theta + \frac{m\pi}{4}\right)\cos\left(\theta + \frac{(m-1)\pi}{4}\right) \\ -\cos\left(\theta + \frac{m\pi}{4}\right)\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\end{array}\right]}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4$$

$$\Rightarrow \sum_{m=1}^{6}\left[\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right] = 4$$

$$\Rightarrow \left[\cot\theta - \cot\left(\theta + \frac{\pi}{4}\right)\right] + \left[\cot\left(\theta + \frac{\pi}{4}\right) - \cot\left(\theta + \frac{2\pi}{4}\right)\right]$$

$$+ ... + \left[\cot\left(\theta + \frac{5\pi}{4}\right) - \cot\left(\theta + \frac{6\pi}{4}\right)\right] = 4$$

$$\Rightarrow \cot\theta - \cot\left(\theta + \frac{3\pi}{2}\right) = 4 \Rightarrow \cot\theta + \tan\theta = 4$$

$\Rightarrow \cos^2\theta + \sin^2\theta = 4\sin\theta\cos\theta$

$\Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{12} \text{ or } \frac{5\pi}{12}$

**28.** **(a, b)** The given hyperbola is

$x^2 - y^2 = \frac{1}{2}$ ...(1)

which is a rectangular hyperbola (i.e. $a = b$)

$\therefore \quad e = \sqrt{2}.$

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Its eccentricity $= \frac{1}{\sqrt{2}}$

$\therefore \quad b^2 = a^2\left(1 - \frac{1}{2}\right) \quad \Rightarrow \quad b^2 = \frac{a^2}{2}$

So, the equation of ellipse becomes

$x^2 + 2y^2 = a^2$ ...(2)

Let the hyperbola (1) and ellipse (2) intersect each other at $P(x_1, y_1)$.

Then slope of hyperbola (1) at $P$ is given by

$m_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{x_1}{y_1}$

and that of ellipse (2) at $P$ is

$m_2 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = -\frac{x_1}{2y_1}$

As the two curves intersect orthogonally,

$\therefore \quad m_1 m_2 = -1$

$\Rightarrow \frac{x_1}{y_1}.\left(-\frac{x_1}{2y_1}\right) = -1$

$\Rightarrow x_1^2 = 2y_1^2$ ...(i)

Also $P(x_1, y_1)$ lies on $x^2 - y^2 = \frac{1}{2}$

$\therefore \quad x_1^2 - y_1^2 = \frac{1}{2}$ ...(ii)

Solving (i) and (ii), we get $y_1^2 = \frac{1}{2}$ and $x_1^2 = 1$

Also $P(x_1, y_1)$ lies on ellipse $x^2 + 2y^2 = a^2$

$\therefore \quad x_1^2 + 2y_1^2 = a^2 \Rightarrow 1 + 1 = a^2$ or $a^2 = 2$

$\therefore$ The required ellipse is $x^2 + 2y^2 = 2$ whose foci are $(\pm ae, 0) = \left(\pm\sqrt{2} \times \frac{1}{\sqrt{2}}, 0\right) = (\pm 1, 0)$

**29.** ***A q,s; B p,r,s,t; C t; D r***

(A) The given equation is

$2\sin^2\theta + \sin^2 2\theta = 2$

$\Rightarrow 2\sin^2\theta + 4\sin^2\theta\cos^2\theta - 2 = 0$

$\Rightarrow \sin^2\theta + 2\sin^2\theta(1 - \sin^2\theta) - 1 = 0$

$\Rightarrow 2\sin^4\theta - 3\sin^2\theta + 1 = 0$

$\Rightarrow 2\sin^4\theta - 2\sin^2\theta - \sin^2\theta + 1 = 0$

$\Rightarrow 2\sin^2\theta(\sin^2\theta - 1) - 1(\sin^2\theta - 1) = 0$

$\Rightarrow (\sin^2\theta - 1)(2\sin^2\theta - 1) = 0$

$\Rightarrow \sin^2\theta = 1$ or $\sin^2\theta = \frac{1}{2}$

$\Rightarrow \sin^2\theta = \sin^2\frac{\pi}{2}$ or $\sin^2\theta = \sin^2\frac{\pi}{4}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{2}$ or $n\pi \pm \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2}$ or $\frac{\pi}{4}$

(B) We know that $[x]$ is discontinuous at all integral values, therefore $\left[\frac{6x}{\pi}\right]$ is discontinuous at $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ and $\pi$. Also $\cos\left[\frac{3x}{\pi}\right] \neq 0$ for any of these values of $x$.

$\therefore \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$ is discontinuous at $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ and $\pi$.

(C) We know that the volume of a parallelopipe with coterminus edges as $\vec{a}, \vec{b}$ and $\vec{c}$ is given by $[\vec{a}\ \vec{b}\ \vec{c}]$

$\therefore$ The required volume is $= \vec{a}.\vec{b} \times \vec{c}$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi$$

(D) We have $\vec{a} + \vec{b} = -\sqrt{3}\vec{c} \Rightarrow |\vec{a} + \vec{b}|^2 = 3|\vec{c}|^2$

$\Rightarrow (\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = 3\vec{c}.\vec{c}$

$\Rightarrow \vec{a}.\vec{a} + \vec{b}.\vec{b} + 2\vec{a}.\vec{b} = 3\vec{c}.\vec{c} \Rightarrow 1 + 1 + 2\cos\theta = 3$

(where $\theta$ is the angle between $\vec{a}$ and $\vec{b}$)

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

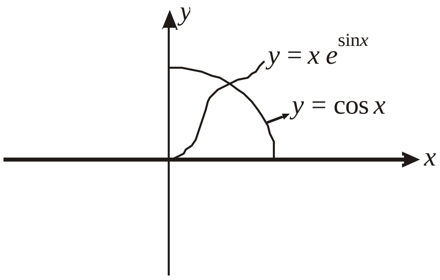
**30.** ***A p; B q,s; C q,r,s,t; D r***

(A) For the solution of $xe^{\sin x} - \cos x = 0$ in $\left(0, \frac{\pi}{2}\right)$

Let us consider two functions

$y = xe^{\sin x}$ and $y = \cos x$

The range of $y = xe^{\sin x}$ is $\left(0, \frac{\pi e}{2}\right)$, also it is an increasing function on $\left(0, \frac{\pi}{2}\right)$. Their graph are as shown in the figure below :



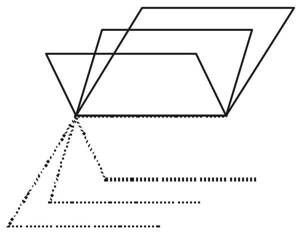

Clearly the two curves meet only at one point, therefore the given equation has only one solution in $\left(0, \frac{\pi}{2}\right)$.

(B) Three given planes are

$kx + 4y + z = 0$

$4x + ky + 2z = 0$

$2x + 2y + z = 0$



Clearly all the planes pass through (0,0,0).

$\therefore$ Their line of intersection also pass through (0, 0, 0)

Let a, b, c, be the direction ratios of required line, then we should have

$ka + 4b + c = 0$

$4a + kb + 2c = 0$

$2a + 2b + c = 0$

For the required line to exist the above system of equations in *a*, *b*, *c*, should have non trivial solution i.e.

$$\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad k(k-4) - 4(4-4) + 1(8-2k) = 0$

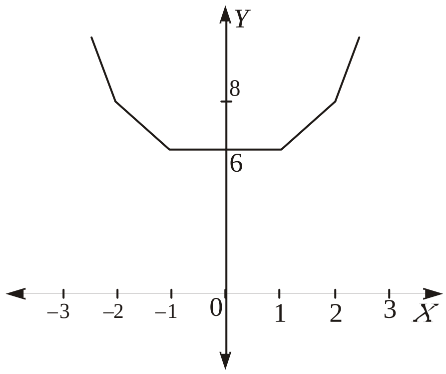
$\Rightarrow \quad k^2 - 6k + 8 = 0 \Rightarrow (k-2)(k-4) = 0$

$\Rightarrow \quad k = 2 \text{ or } 4$

(C) We have $f(x) = |x-1| + |x-2| + |x+1| + |x+2|$

$$= \begin{cases} -4x, & x \le -2 \\ -2x+4, & -2 < x \le -1 \\ 6, & -1 < x \le 1 \\ 2x+4, & 1 < x \le 2 \\ 4x, & x \ge 2 \end{cases}$$

The graph of the above function is as given below




Clearly, from graph, $f(x) \ge 6$

$\Rightarrow \quad 4k \ge 6 \quad \Rightarrow \quad k \ge \frac{3}{2}$

$\therefore \quad k = 2, 3, 4, 5, 6, .....$

(D) Given that

$\frac{dy}{dx} = y + 1$ and $y(0) = 1$

$\Rightarrow \quad \int \frac{dy}{y+1} = \int dx \quad \Rightarrow \quad \ln|y+1| = x + c$

At $x = 0, y = 1 \Rightarrow c = \ln 2$

$\therefore \ln|y+1| = x + \ln 2 \quad \Rightarrow \quad y + 1 = 2e^x \Rightarrow \quad y = 2e^x - 1$

$\therefore \quad y(\ln 2) = 2e^{\ln 2} - 1 = 2 \times 2 - 1 = 3$

**31. Integer answer : 8**

Let *r* be the radius of required circle.

Clearly, in $\Delta C_1CC_2$, $C_1C = C_2C = r + 1$

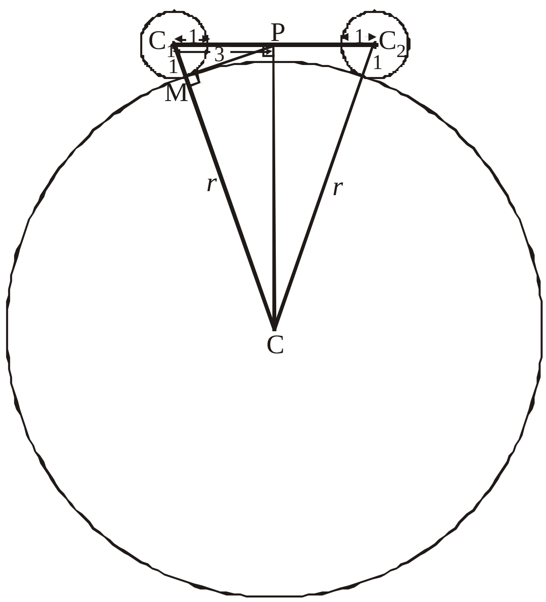
and *P* is mid point of $C_1C_2$

$\therefore \quad CP \perp C_1C_2$

Also $PM \perp CC_1$

Now $\Delta PMC_1 \sim \Delta CPC_1$ (by AA similarity)

$\therefore \quad \frac{MC_1}{PC_1} = \frac{PC_1}{CC_1}$



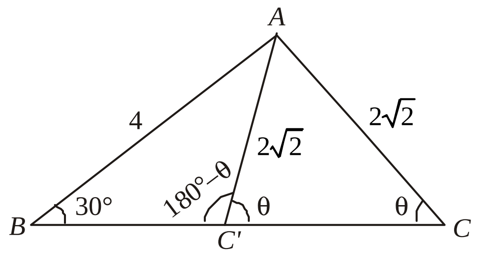
$\Rightarrow \quad \frac{1}{3} = \frac{3}{r+1} \Rightarrow r+1 = 9 \Rightarrow r = 8.$

**32. Integer answer : 4**

Let $\angle ACC' = \theta$

then $\angle AC'C = \theta \quad (\because \; AC = AC')$

and $\angle AC'B = 180 - \theta.$




Applying sine law in $\Delta ABC'$, we get

$$\frac{4}{\sin(180-\theta)} = \frac{2\sqrt{2}}{\sin 30°}$$

$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

$\therefore \quad \angle CAC' = 90°$

So, the required area $= ar(\Delta ABC) - ar(\Delta ABC')$

$$= ar(\Delta ACC') = \frac{1}{2} \times AC \times AC'$$

$$= \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} = 4 \text{ sq. units.}$$

**33. Integer answer : 0**

Given that $f(x) = \int_0^x f(t)\,dt$

Clearly $f(0) = 0$. Also $f'(x) = f(x)$

$\Rightarrow \quad \frac{f'(x)}{f(x)} = 1$

Integrating both sides with respect to $x$, we get

$$\int \frac{f'(x)}{f(x)} dx = \int 1\, dx$$

$\Rightarrow \quad \ln f(x) = x + \ln C \quad \Rightarrow f(x) = Ce^x$

Now $f(0) = 0 \Rightarrow Ce^x = 0 \Rightarrow C = 0$

$\therefore \quad f(x) = 0 \; \forall \; x \Rightarrow f(\ln 5) = 0$

**34. Integer answer : 7**

The given system of equations is

$3x - y - z = 0$

$-3x + z = 0$

$-3x + 2y + z = 0$

Let $x = p$ where $p$ is an integer
then $y = 0$ and $z = 3p$

But $x^2 + y^2 + z^2 \le 100 \Rightarrow p^2 + 9p^2 \le 100$

$\Rightarrow \quad p^2 \le 10 \Rightarrow p = 0, \pm 1, \pm 2 \pm 3$

i.e. $p$ can take 7 different values.

$\therefore$ Number of points (x, y, z) are 7.

**35. Integer answer : 2**

Given that $f(x) = x^3 + e^{x/2}$

Let $g(x) = f^{-1}(x)$

then we should have $gof(x) = x$

$\Rightarrow \quad g(f(x)) = x$

$\Rightarrow \quad g(x^3 + e^{x/2}) = x$

Differentiating both sides with respect to $x$, we get

$$g'(x^3 + e^{x/2}).\left(3x^2 + e^{x/2}.\frac{1}{2}\right) = 1$$

$$\Rightarrow \quad g'(x^3 + e^{x/2}) = \frac{1}{3x^2 + e^{x/2}.\frac{1}{2}}$$

For $x = 0$, we get $g'(1) = \frac{1}{1/2} = 2$

**36. Integer answer : 2**

The given equation is

$$x^2 - 8kx + 16(k^2 - k + 1) = 0$$

$\because$ Both the roots are real and distinct

$\therefore \quad D > 0 \Rightarrow (8k)^2 - 4 \times 16(k^2 - k + 1) > 0$

$\Rightarrow \quad k > 1$ ...(i)

$\because$ Both the roots are greater than or equal to 4

$\therefore \quad \alpha + \beta > 8$ and $f(4) \ge 0$

$\Rightarrow \quad k > 1$ ...(ii)

and $16 - 32k + 16(k^2 - k + 1) \ge 0$

$\Rightarrow \quad k^2 - 3k + 2 \ge 0 \Rightarrow (k-1)(k-2) \ge 0$

$\Rightarrow \quad k \in (-\infty, 1] \cup [2, \infty)$ ...(iii)

Combining (i), (ii) and (iii), we get $k \ge 2$ or the smallest value of $k = 2$.

**37. Integer answer : 7**

The given function is $f(x) = 2x^3 - 15x^2 + 36x - 48$

and $A = \{x \mid x^2 + 20 \le 9x\}$

$\Rightarrow \quad A = \{x \mid x^2 - 9x + 20 \le 0\}$

$\Rightarrow \quad A = \{x \mid (x-4)(x-5) \le 0\}$

$\Rightarrow \quad A = [4, 5]$

Also $f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$

$= 6(x-2)(x-3)$

Clearly $\forall \; x \in A, \; f'(x) > 0$

$\therefore$ $f$ is strictly increasing function on $A$.

$\therefore$ Maximum value of $f$ on $A$

$= f(5) = 2 \times 5^3 - 15 \times 5^2 + 36 \times 5 - 48$

$= 250 - 375 + 180 - 48 = 430 - 423 = 7.$

**38. Integer answer : 0**

Let $p(x) = ax^4 + bx^3 + cx^2 + dx + e$

Now $\lim_{x \to 0}\left[1 + \frac{p(x)}{x^2}\right] = 2$

$\Rightarrow \quad \lim_{x \to 0} \frac{p(x)}{x^2} = 1$ ...(1)

$\Rightarrow \quad p(0) = 0 \Rightarrow e = 0$

Applying L'Hospital's rule to $eq^n$ (1), we get

$$\lim_{x\to 0}\frac{p'(x)}{2x}=1 \Rightarrow p'(0)=0$$

$\Rightarrow \quad d=0$

Again applying L 'Hospital's rule, we get

$$\lim_{x\to 0}\frac{p''(x)}{2}=1 \Rightarrow p''(0)=2$$

$\Rightarrow \quad 2c=2$ or $c=1$

$\therefore \quad p(x)=ax^4+bx^3+x^2$

$\Rightarrow \quad p'(x)=4ax^3+3bx^2+2x$

As $p(x)$ has extremum at $x=1$ and 2

$\therefore \quad p'(1)=0$ and $p'(2)=0$

$\Rightarrow \quad 4a+3b+2=0$ ...(i)

$\Rightarrow \quad 32a+12b+4=0$ or $8a+3b+1=0$ ...(ii)

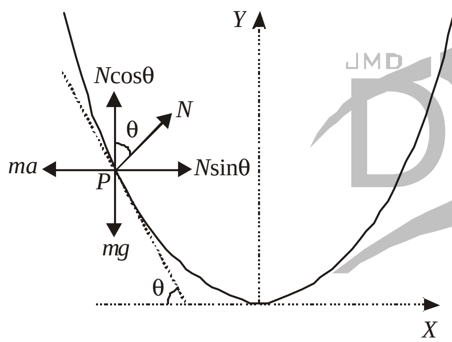
Solving eq's (i) and (ii) we get $a=\frac{1}{4}$ and $b=-1$

$\therefore \quad p(x)=\frac{1}{4}x^4-x^3+x^2$

So, that $p(2)=\frac{16}{4}-8+4=0$

## PHYSICS

**39. (b)**




The forces acting on the bead as seen by the observer in the accelerated frame are : (a) $N$ ; (b) $mg$ ; (c) $ma$ (pseudo force).

Let θ is the angle which the tangent at $P$ makes with the $X$- axis. As the bead is in equilibrium with respect to the wire, therefore

$N\sin\theta = ma$ and $N\cos\theta = mg$

$\therefore \quad \tan\theta=\frac{a}{g}$ … (i)

But $y=kx^2$. Therefore,

$$\frac{dy}{dx}=2kx=\tan\theta \quad \text{… (ii)}$$

From (i) & (ii)

$$2kx=\frac{a}{g} \Rightarrow x=\frac{a}{2kg}$$

**40. (a)** The energy possessed by photons of wavelength 550 nm is $\frac{1240}{550}=2.25\text{eV}$

The energy possessed by photons of wavelength 450 nm is $\frac{1240}{450}=2.76\text{eV}$

The energy possessed by photons of wavelength 350 nm is $\frac{1240}{350}=3.54\text{eV}$

**For metal plate *p* :** $\phi_p = 2\ eV$.

All the wavelengths are capable of ejecting electrons. Therefore, the current is maximum. Also as the work function is lowest in $p$, the kinetic energy of ejected electron will be highest and therefore, the stopping potential is highest.

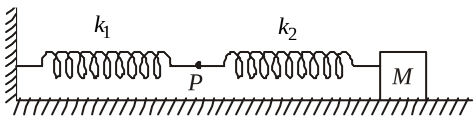
**For metal plate *q* :** $\phi_q = 2.5\ eV$.

Photons of wavelength 550 nm will not be able to eject electrons and therefore, the current is smaller than $p$. The work function is greater than $q$ therefore the stopping potential is lower in comparison to $p$.

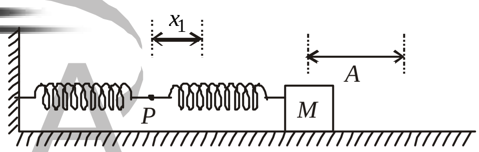
**For metal plate *r* :** $\phi_r = 3\ eV$

Only wavelength of 350 nm will be able to eject electrons and therefore, current is minimum. Also the stopping potential is least.

**41. (d)**




Case (i)




Case (ii)

In case (ii), the springs are shown in the maximum compressed position. If the spring of spring constant $k_1$ is compressed by $x_1$ and that of spring constant $k_2$ is compressed by $x_2$ then

$x_1+x_2=A$ … (i)

and $k_1x_1=k_2x_2 \Rightarrow x_2=\frac{k_1x_1}{x_2}$ …(ii)

From (i) & (ii)

$$x_1+\frac{k_1x_1}{k_2}=A \Rightarrow x_1=\frac{k_2A}{k_2+k_1}$$

**42. (c)**

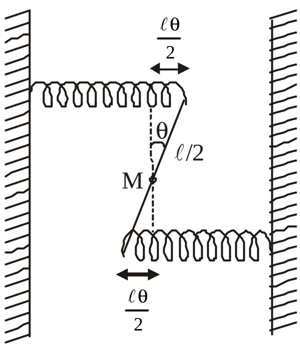



Figure shows the rod at an angle θ with respect to its equilibrium position. Both the springs are stretched by length $\frac{\ell\theta}{2}$.

The restoring torque due to one spring
= – (Restoring force) × perpendicular distance

$$= -k\left(\frac{\ell\theta}{2}\right)\times\frac{\ell}{2} = -k\frac{\ell^2}{4}\theta$$

Therefore, the total restoring torque due to both the springs $= 2\times\left[-k\frac{\ell^2}{4}\theta\right] = -k\frac{\ell^2}{2}\theta$ … (i)

If $I$ is the moment of inertia of the rod about $M$ then

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} \quad \text{… (ii)}$$

From (i) & (ii), we get

$$I\frac{d^2\theta}{dt^2} = -k\frac{\ell^2\theta}{2} \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{k}{I}\frac{\ell^2}{2}\theta = \frac{-k}{M\ell^2/12}\frac{\ell^2}{2}\theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{6k}{M}\theta$$

Comparing it with the standared equation of rotational SHM we get

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \Rightarrow \omega^2 = \frac{6k}{M} \Rightarrow \omega = \sqrt{\frac{6k}{M}}$$

$$\Rightarrow 2\pi v = \sqrt{\frac{6k}{M}} \Rightarrow v = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}.$$

**43. (b, d) For process B $\rightarrow$ C $\rightarrow$ D :**

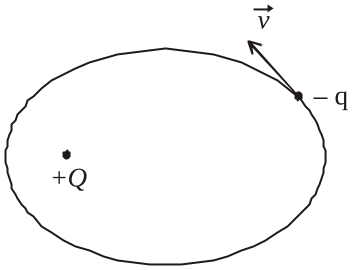
The temperature at B is greater than the temperature at D. Therefore, $\Delta U$ = –ve. Also work done in the process is negative. Thus heat flows out of the gas during the process.

**For the cyclic process ABCDA :**

As the process takes place in the clockwise direction. Therefore the work done is positive.

- Process during the path A $\rightarrow$ B is circular and therefore the process is not isothermal (for isothermal process, the path in P-V graph is a rectangular hyperbola).
- During A $\rightarrow$ B $\rightarrow$ C, the work done is positive.

**44. (a)**

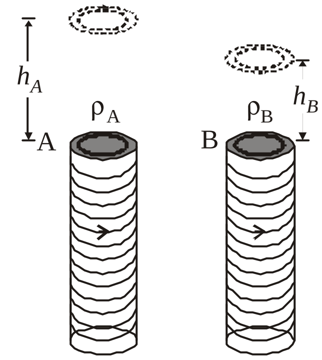



The situation is shown in the figure which is similar to a planet revolving around sun. The distance of – $q$ from + $Q$ is changing, therefore, force between the charges will change.

The speed of the charge – $q$ will be greater when the charge is nearer to + $Q$ as compared to when it is far. Therefore, the angular velocity of charge – $q$ is also variable. The direction of the velocity changes continously, therefore, linear momentum is also variable. The angular momentum of (– $q$) about $Q$ is constant because the torque about + $Q$ is zero.

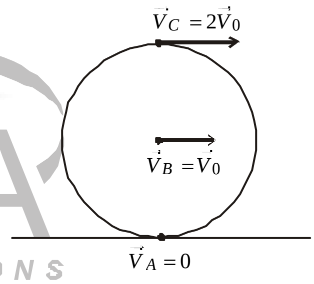
**45. (b, d)**

When current $I$ is switched on in both the solenoids in identical manner, eddy currents are setup in metallic rings $A$ and $B$ in such a way that rings $A$ and $B$ are repelled.




Given $h_A > h_B$. This shows that eddy currents produced in A are greater than in $B$. This is possible when $\rho_A < \rho_B$ (the rate of change of flux is same in both the rings, therefore induced emf is same). The height attained is independent of the masses of rings $A$ and $B$.

**46. (b, c)**




If $\vec{V}_0$ is the velocity of centre of the sphere, then

$\vec{V}_C = 2\vec{V}_0$, $\vec{V}_B = \vec{V}_0$ and $\vec{V}_A = 0$

$$\therefore \quad \vec{V}_C - \vec{V}_B = 2\vec{V}_0 - \vec{V}_0 = \vec{V}_0$$

$$\vec{V}_B - \vec{V}_A = \vec{V}_0 - \vec{0} = \vec{V}_0$$

$$\therefore \quad \vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$$

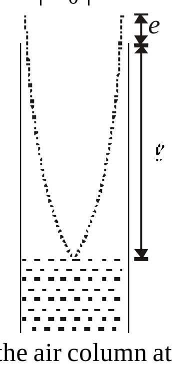
(b) is the correct option.

Now, $|\vec{V}_C - \vec{V}_A| = |2\vec{V}_0 - 0| = |2\vec{V}_0| = 2|\vec{V}_0|$

and $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(c) is the correct option.

**47. (a, d)** At second resonance the length of air column is more as compared to first resonance. Now, longer the length of air column, more is the absorption of energy and lesser is the intensity of sound heard.




As shown in the figure, the length of the air column at the first resonance is somewhat shorter than $\frac{1}{4}$th of the wavelength of the sound in air due to end correction.

$\ell + e = \frac{\lambda}{4}$ $\quad\therefore\quad \ell = \frac{\lambda}{4} - e$

**48.** ***A → (p,q,t); B → (q); C → (s); D → (s)***

(p) When an uncharged capacitor is connected to a battery, it becomes charged and energy is stored in the capacitor. (A) is the correct option.

(q) When a gas in an adiabatic container fitted with an adiabatic piston is compressed by pushing the piston

(i) the internal energy of the system increases

$$\Delta U = Q - W = 0 - (-PdV) = +PdV$$

(ii) mechanical energy is proceeded to the piston which is converted into kinetic energy of the gas molecules.

(r) None of the options in column I matches. As the gas in a rigid container gets cooled, the internal energy of the system will decrease. The average kinetic energy per molecule will decrease.

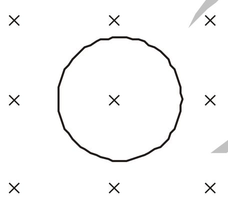
(s) When a heavy nucleus initially at rest splits into two nuclei of nearly equal masses and some neutrons are emitted then

(i) internal energy of the system is converted into mechanical energy (precisely speaking kinetic energy) and

(ii) mass of the system decreases which converts into energy.

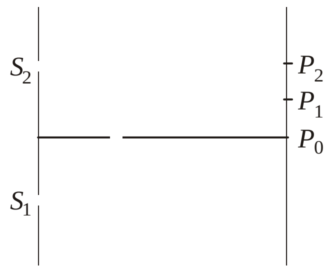
(t) When a resistive wire loop is placed in a time varying magnetic field perpendicular to its palne.

(i) Induced current shows in the loop due to which the energy of system is increased.



**49.** ***A → (p,s); B → (q); C → (t); D → (r,s,t)***

***(A)***




For path difference $\lambda/4$, phase difference is $\pi/2$.

For path difference $\lambda/3$, phase difference is $2\pi/3$.

Here, $S_1P_0 - S_2P_0 = 0$

$\therefore \quad \delta(P_0) = 0$

Therefore, (*p*) matches with (*A*).

The path difference for $P_1$ and $P_2$ will not be zero. The intensities at $P_0$ is maximum.

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos 0^\circ$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{I_0} + \sqrt{I_0})^2 = 4I_0$$

$$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\frac{\pi}{2}$$

$$= I_1 + I_2 = I_0 + I_0 = 2I_0$$

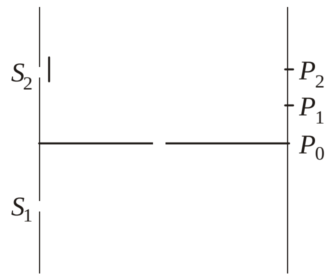
$$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(2\pi/3)$$

$$= I_1 + I_2 - \sqrt{I_1}\sqrt{I_2} = I_0 + I_0 - I_0 = I_0$$

$\therefore \quad I(P_0) > I(P_1)$

Therefore, (*s*) matches with (*A*).

(B)




$\delta(P_0) = \frac{\lambda}{4}$, $\delta(P_1) = 0$, $\delta(P_2) = \frac{\lambda}{12}$

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\pi/2$$

$$= I_1 + I_2 = I_0 + I_0 = 2I_0$$

$$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} = 4I_0$$

$$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\pi/6$$

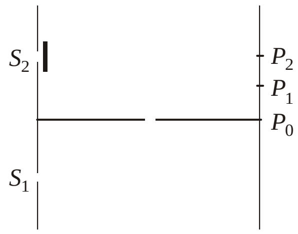
$$= I_1 + I_2 + \sqrt{3}\sqrt{I_1}\sqrt{I_2}$$

$$= I_0 + I_0 + \sqrt{3}I_0$$

$$= (2 + \sqrt{3})I_0$$

Therefore, q matches with (B)

(C)




Here $\delta(P_0) = -\lambda/2$; $\delta(P_1) = -\lambda/4$, $\delta(P_2) = -\lambda/6$

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(-\pi)$$

$$= I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2} = I_0 + I_0 - 2I_0 = 0$$

$$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(-\pi/2)$$

$$= I_1 + I_2 = I_0 + I_0 = 2I_0$$

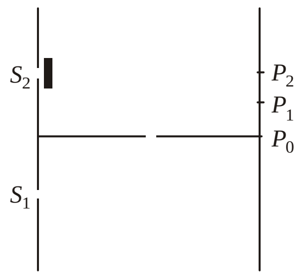
$$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\left(-\frac{\pi}{3}\right)$$

$$= I_1 + I_2 + \sqrt{I_1}\sqrt{I_2} = I_0 + I_0 + I_0 = 3I_0$$

$\therefore \quad I(P_2) > I(P_1)$

(*t*) matches (*C*).

(D)




Here, $\delta(P_0) = 3\lambda/4; \delta(P_1) = -\lambda/2; \delta(P_2) = -5\lambda/12$

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\left(\frac{-3\pi}{2}\right)$$

$$= I_1 + I_2 = I_0 + I_0 = 2I_0$$

$$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(-\pi)$$

$$= I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2} = I_0 + I_0 - 2\sqrt{I_0}\sqrt{I_0} = 0$$

$$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos[-5\pi/6]$$

$$= I_1 + I_2 - \sqrt{3}\sqrt{I_1}\sqrt{I_2} = \left(2-\sqrt{3}\right)I_0$$

$\therefore$ (*r*), (*s*), (*t*) match with (*D*).

**50. (8 J)**

Given $m = 0.36$ kg, $M = 0.72$ kg.

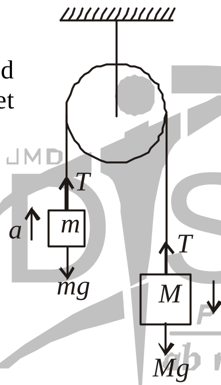
The figure shows the forces on $m$ and $M$. When the system is released, let the acceleration be $a$. Then

$T - mg = ma$

$Mg - T = Ma$

$\therefore \quad a = \frac{(M-m)g}{M+m} = g/3$

and $T = 4mg/3$




**For block m :**

$u = 0, a = g/3, t = 1, s = ?$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times\frac{g}{3}\times 1^2 = g/6$$

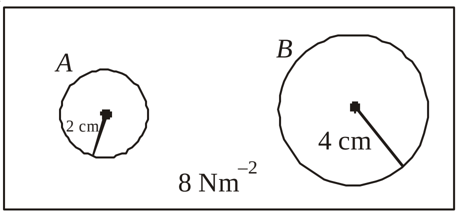
$\therefore$ Work done by the string on $m$ is

$$\vec{T}.\vec{s} = Ts = 4\frac{mg}{3}\times\frac{g}{6} = \frac{4\times 0.36\times 10\times 10}{3\times 6} = 8\,\text{J}$$

**51. (6)**

**For bubble *A* :**

If $P_A$ is the pressure inside the bubble then




$$P_A - 8 = \frac{4T}{R_A} = \frac{4\times 0.04}{0.02} = 8 \Rightarrow P_A = 16N/m^2$$

According to ideal gas equation,

$$P_A V_A = n_A R T_A \Rightarrow 16\times\frac{4}{3}\pi(0.02)^3 = n_A R T_A \ \ldots \text{(i)}$$

**For bubble *B* :**

If $P_B$ is the pressure inside the bubble then

$$P_B - 8 = \frac{4T}{R_B} = \frac{4\times 0.04}{0.04} = 4 \Rightarrow P_B = 12N/m^2$$

According to ideal gas equation,

$$P_B V_B = n_B R T_B \Rightarrow 12\times\frac{4}{3}\pi(0.04)^3 = n_B R T_B \ \ldots \text{(ii)}$$

Dividing (ii) by (i) we get

$$\frac{12\times\frac{4}{3}\pi(0.04)^3}{16\times\frac{4}{3}\pi(0.02)^3} = \frac{n_B}{n_A} \quad [\because T_A = T_B]$$

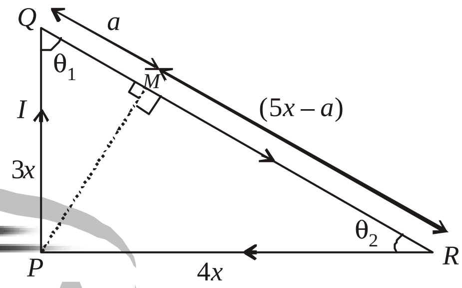
$$\therefore \quad \frac{n_B}{n_A} = 6$$

**52. (k = 7)**

The right angled triangle is shown in the figure. Let us drop a perpendicular from $P$ on $QR$ which cuts $QR$ at $M$.

The magnatic field due to currents in $PQ$ and $RP$ at $P$ is zero.

The magnetic field due to current in $QR$ at $P$ is




$$B = \frac{\mu_0}{4\pi}\frac{I}{PM}(\cos\theta_1 + \cos\theta_2) \quad \ldots\text{(i)}$$

In ΔPQM,

$$9x^2 = PM^2 + a^2 \quad \ldots\text{(ii)}$$

In ΔPRM,

$$16x^2 = PM^2 + (5x-a)^2 \quad \ldots\text{(iii)}$$

$$\Rightarrow \quad 7x^2 = 25x^2 - 10xa \Rightarrow 10xa = 18x^2$$

$$\Rightarrow \quad a = 1.8x \quad \ldots\text{(iv)}$$

From (ii) & (iv),

$$9x^2 = PM^2 + (1.8x)^2$$

$$\Rightarrow \quad PM = \sqrt{9x^2 - 3.24x^2} = \sqrt{5.76x^2} = 2.4x \ \ldots\text{(v)}$$

Also $\cos\theta_1 = \frac{a}{3x} = \frac{1.8x}{3x} = 0.6 \quad \ldots\text{(vi)}$

$$\cos\theta_2 = \frac{5x-a}{4x} = \frac{5x-1.8x}{4x} = \frac{3.2}{4} = 0.8 \quad \ldots\text{(vii)}$$

From (i), (v), (vi) and (vii),

$$B = \frac{\mu_0}{4\pi}\times\frac{I}{2.4x}[0.6+0.8]$$

$$= \frac{\mu_0}{4\pi}\times\frac{I}{2.4x}\times 1.4 = 7\left[\frac{\mu_0 I}{48\pi x}\right]$$

Comparing it with B = $k\left[\frac{\mu_0 I}{48\pi x}\right]$, we get, k = 7.

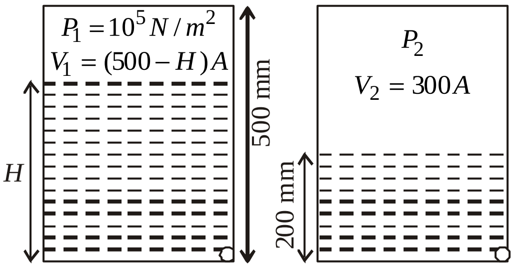
**53. (6 mm)**

Initially, the pressure of air column above water is $P_1 = 10^5$ Nm$^{-2}$ and volume $V_1 = (500 - H)A$, where $A$ is the area of cross-section of the vessel.

Finally, the volume of air column above water is 300 A. If $P_2$ is the pressure of air then

$$P_2 + \rho g h = 10^5$$

$$\therefore \quad P_2 + 10^3 \times 10 \times \frac{200}{1000} = 10^5$$




Initially Finally

$$\therefore \quad P_2 = 9.8 \times 10^4 \, N/m^2$$

As the temperature remains constant, according to Boyle's law

$$P_1V_1 = P_2V_2$$

$$10^5 \times (500 - H)A = (9.8 \times 10^4) \times 300A$$

$\Rightarrow$ H = 206 mm

$\therefore$ The fall of height of water level due to the opening of orifice = 206 – 200 = 6 mm

**54. (5 cm)**

We know that, $\nu = \sqrt{\frac{T}{\mu}}$

where $T$ = tension in the string and $\mu = \frac{\text{mass}}{\text{length}}$

$$v = \sqrt{\frac{0.5}{10^{-3}/0.2}} = 10 m/s$$

The wavelength of the wave established

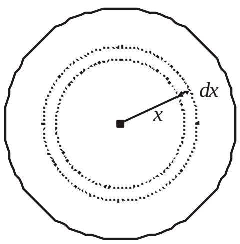
$$\lambda = \frac{v}{f} = \frac{10}{100} = 0.1m = 10cm$$

$\therefore$ The distance between two successive nodes

$$= \frac{\lambda}{2} = \frac{10}{2} = 5cm$$

**55. (a = 2)**

Let us consider a spherical shell of radius $x$ and thickness $dx$. The volume of this shell is $4\pi x^2(dx)$. The charge enclosed in this spherical shell is




$$dq = (4\pi x^2)dx \times kx^a$$

$$\therefore \quad dq = 4\pi k x^{2+a} dx.$$

**For r = R :**

The total charge enclosed in the sphere of radius $R$ is

$$Q = \int_0^R 4\pi k \, x^{2+a} \, dx = 4\pi k \frac{R^{3+a}}{3+a}.$$

$\therefore$ The electric field at $r = R$ is

$$E_1 = \frac{1}{4\pi\varepsilon_0} \frac{4\pi k R^{3+a}}{(3+a)R^2} = \frac{1}{4\pi\varepsilon_0} \frac{4\pi k}{3+a} R^{1+a}$$

**For r = R/2 :**

The total charge enclosed in the sphere of radius $R/2$ is

$$Q' = \int_0^{R/2} 4\pi k \, x^{2+a} dx = \frac{4\pi k (R/2)^{3+a}}{3+a}$$

$\therefore$ The electric field at $r = R/2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0} \frac{4\pi k}{3+a} \frac{(R/2)^{3+a}}{(R/2)^2} = \frac{1}{4\pi\varepsilon_0} \frac{4\pi k}{3+a} \left(\frac{R}{2}\right)^{1+a}$$

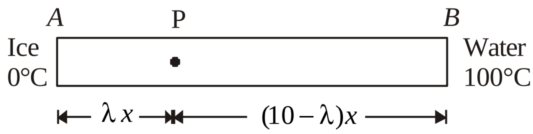
Given, $E_2 = \frac{1}{8} E_1$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0} \frac{4\pi k}{(3+a)} \left(\frac{R}{2}\right)^{1+a} = \frac{1}{2^3} \times \frac{1}{4\pi\varepsilon_0} \frac{4\pi k}{3+a} R^{1+a}$$

$\Rightarrow \quad 1 + a = 3 \Rightarrow a = 2$

**56. (l = 9)**

**Heat flow from *P* to *A* :**




The heat flow per unit time

$$Q = \frac{KA(400-0)}{\lambda x} = mL_{fus} \qquad \dots (1)$$

where $K$ = thermal conductivity of rod and $A$ its area of cross-section, m = mass of ice melting /sec.

**Heat flow from *P* to *B* :**

The heat flows per unit time

$$Q = \frac{KA(400-100)}{(10-\lambda)x} = mL_{vap} \qquad \dots (ii)$$

$m$ = mass of water evaporated/sec

Dividing (1) by (ii), we get

$$\frac{400(10-\lambda)}{\lambda \times 300} = \frac{L_{fus}}{L_{vap}} = \frac{80}{540}$$

$$\Rightarrow \quad \frac{4(10-\lambda)}{3\lambda} = \frac{4}{27}$$

$\Rightarrow \quad 90 - 9\lambda = \lambda$

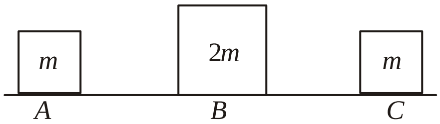
$\Rightarrow \quad 10\lambda = 90$

$\Rightarrow \quad \lambda = 9$

**57. (4 m/s)**

The velocity of $B$ just after collision with $A$ is

$$v_B = \frac{(m_B - m_A)u_B}{m_B + m_A} + \frac{2m_A u_A}{m_A + m_B}$$




$$= \frac{0 + 2m \times 9}{m + 2m} = 6m/s$$

The collision between $B$ and $C$ is completely inelastic.

$\therefore \quad m_B v_B = (m_B + m_c) v$

$$\therefore \quad v = \frac{6 \times 2m}{2m + m} = 4\text{m/s}.$$