SOLUTIONS

1. (c) As Sb_2S_3 is a negative sol, so $Al_2(SO_4)_3$ will be the most effective coagulant due to higher positive charge on Al (Al^{3+}) – **Hardy-Schulze rule**.

2. (b) Average atomic mass of Fe

$$= \frac{(54\times5)+(56\times90)+(57\times5)}{100} = 55.95$$

3. (a) Carboxylic acid is stronger acid than phenol. The presence of electron withdrawing group (e.g. Cl) increases acidic strength, while presence of electron donating group (e.g. CH_3) decreases acidic strength.

4. (b) – CN has highest priority. Further the sum of locants is 7 in (b) and 9 in (d).

5. (b) Correction factor for attractive force for *n* moles of real gas is given by the term mentioned in (b).

6. (d) Nylon and cellulose, both have intermolecular hydrogen bonding, polyvinyl chloride has dipole-dipole interaction, while natural rubber has van der Waal forces which are weakest.

7. (b) $P_4 + 3O_2 \xrightarrow{\text{in presence of } N_2} P_4O_6$

Here N_2 acts as a diluent and thus retards further oxidation. Reaction of P_4 under other three conditions.

(a) $P_4 + 3O_2 \longrightarrow P_4O_6 \xrightarrow{2O_2} P_4O_{10}$

(c) In moist air, P_4O_6 is hydrolysed to form H_3PO_3

$$P_4O_6 + 6H_2O \longrightarrow 4H_3PO_3$$

(d) In presence of NaOH,

$$P_4 + 3OH^- + 3H_2O \longrightarrow PH_3 + 3H_2PO_2^-$$

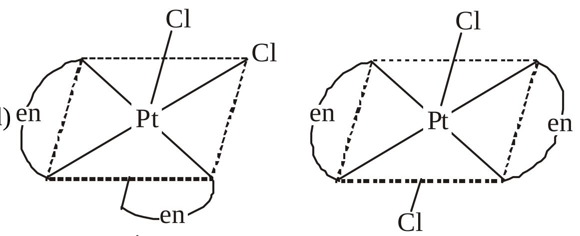
8. (a) $P_{N_2} = \kappa_H \chi_{N_2}$

$0.8 \times 5 = 1 \times 10^5 \times \chi_{N_2}$

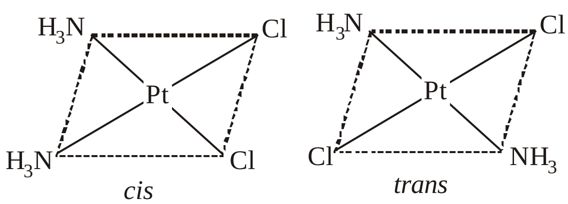
$\therefore \chi_{N_2} = 4 \times 10^{-5}$

Solubility in 10 moles = 4×10^{-4}.

9. (c,d)



cis *trans*



cis *trans*

10. (a,b) $4Na + O_2\,(\text{limited}) \xrightarrow{\Delta} 2Na_2O$

$$2Na + O_2\,(\text{excess}) \xrightarrow{\Delta} Na_2O_2$$

11. (b,c) Frenkel defect is a dislocation effect, observed when the size of the cation and anion differ largely. F-center is created when an anion is lost from the lattice and vacancy is filled by trapping of an electron. Schottky defect changes the density of a crystalline solid.

12. (a,d) $CH_3 - \overset{*}{C}H(OH) - CH = CH - \overset{*}{C}H(OH) - CH_3$

Stereoisomer	Configuration		
I	*d*	*cis*	*d*
II	*l*	*cis*	*l*
III	*d*	*cis*	*l*
IV	*d*	*trans*	*d*
V	*l*	*trans*	*l*
VI	*d*	*trans*	*l*

Enantiomers: I and II (*cis*); IV and V (*trans*)

Diastereomers: I (or II), III (or IV), V and VI

Meso: III and IV

For 13-15

Reaction of Y indicates that it is Fe^{3+} salt.

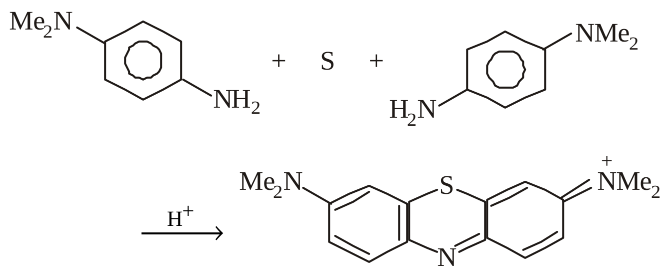
$$\underset{Y}{4Fe^{3+}} + \underset{\text{hexacyanoferrate (II) or ferrocyanide}}{3[Fe(CN)_6]^{4-}} \longrightarrow \underset{\text{Intense blue ppt. soluble in excess of ferrocyanide}}{Fe_4[Fe(CN)_6]_3\downarrow}$$

$$\underset{Y}{Fe^{3+}} + \underset{\text{hexacyanoferrate (III) or ferricyanide}}{[Fe(CN)_6]^{3-}} \longrightarrow \underset{\text{Brown colour}}{Fe[Fe(CN)_6]}$$

Further since the product formed (methylene blue) has sulphur in its structure, it should be supplied by the compound X which is thus Na_2S.

$$Na_2S + 2H^+ \longrightarrow H_2S + 2Na^+$$

$$FeCl_3 + H_2S \longrightarrow FeCl_2 + 2HCl + S$$



Methylene blue

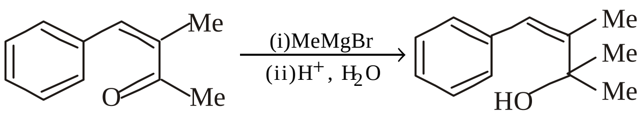
Thus

13. (d) 14. (c) 15. (b)

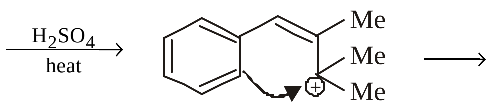
For 16-18

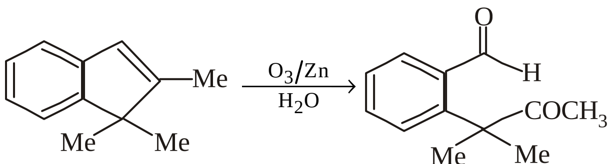
Iodoform test of compound **P** points out that **P** has – $COCH_3$ group which shows that it may be either option (a) or (b) of Q. 16. Further since the dicarbonyl compound **R** has at least one α-H atom w.r.t to one of the carbonyl groups which is possible when **R** is produced from (b) of Q. 18; (a) option of

Q. 16 will give dicarbonyl compound having two –CHO, none of which has α-H atom.

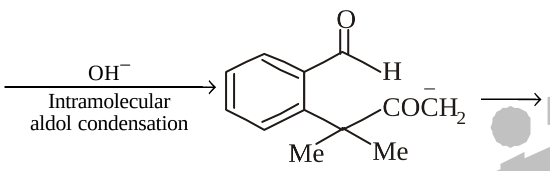


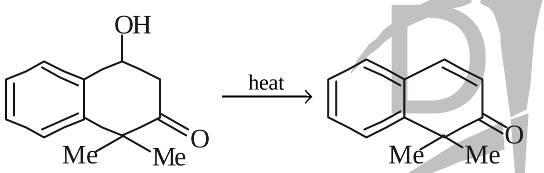
(B)/P





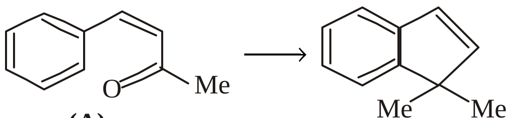
Q **R***



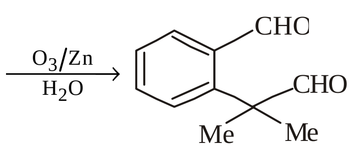


S

* Structure of **R** would be **R'** when **P** is (A)



(A) **Q'**



R'

16. (b) 17. (a) 18. (b)

19. **(A) –p, r, t ; (B) – s,t ; (C) – p, q, r ; (D) – p, r, s**

(A) B_2 $\sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \pi 2p_y^1 = \pi 2p_z^1$

Bond order = 1 Paramagnetic

(B) N_2 $\sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \sigma 2p_x^2, \pi 2p_y^2 = \pi 2p_z^2$

Bond order = 3 Diamagnetic

(C) O_2^- $\sigma 1s^2, \sigma^*1s^2; \sigma 2s^2\ \sigma^*2s^2; \sigma 2p_x^2, \pi 2p_y^2$

$= \pi 2p_z^2,\ \pi^*2p_y^2 = \pi^*2p_z^1$

Bond order = 1.5 Paramagnetic

(D) O_2 $\sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \sigma 2p_x^2, \pi 2p_y^2$

$= \pi 2p_z^2,\ \pi^*2p_y^1 = \pi^*2p_z^1$

Bond order = 2 Paramagnetic

20. **(A)– p, q, s, t; (B) – p, s, t; (C) – p; (D) – r**

(A) $CH_3CH_2CH_2CN \xrightarrow[\text{SnCl}_2/\text{HCl or DIBAL–H}]{\text{Pd–C/H}_2 \text{ or}} CH_3CH_2CH_2CH_2NH_2$

$CH_3CH_2CH_2CN \xrightarrow[H_2O]{OH^-} CH_3CH_2CH_2COO^-$

(B) $CH_3CH_2OCOCH_3 \xrightarrow[\text{DIBAL–H}]{\text{Pd–C/H}_2 \text{ or}} CH_3CH_2OH$

$CH_3CH_2OCOCH_3 \xrightarrow[H_2O]{OH^-} CH_3CH_2OH + CH_3COO^-$

(C) $CH_3CH = CHCH_2OH \xrightarrow{\text{Pd–C/H}_2} CH_3CH_2CH_2CH_2OH$

(D) $CH_3CH_2CH_2CH_2NH_2 \xrightarrow[KOH]{CHCl_3} CH_3CH_2CH_2CH_2NC$

Isonitrile (foul smelling)

MATHEMATICS

21. **(c)** Given that f is a non negative function defined on [0, 1] and

$$\int_0^x \sqrt{1-(f'(t))^2}\, dt = \int_0^x f(t)\, dt, \quad 0 \le x \le 1$$

Differentiating both sides with respect to x, we get

$$\sqrt{1-[f'(x)]^2} = f(x)$$

$$\Rightarrow\ 1-[f'(x)]^2 = [f(x)]^2$$

$$\Rightarrow\ [f'(x)]^2 = 1-[f(x)]^2$$

$$\Rightarrow\ \frac{d}{dx} f(x) = \pm\sqrt{1-[f(x)]^2}$$

$$\Rightarrow\ \pm\frac{d\ f(x)}{\sqrt{1-[f(x)]^2}} = dx$$

Integrating both sides with respect to x, we get

$$\pm\int \frac{d\ f(x)}{\sqrt{1-[f(x)]^2}} = \int dx$$

$\Rightarrow \quad \pm \sin^{-1} f(x) = x + C$

$\because$ Given that $f(0) = 0$

$\Rightarrow \quad C = 0$

Hence $f(x) = \pm \sin x$

But as $f(x)$ is a non negative function on $[0, 1]$

$\therefore \quad f(x) = \sin x.$

Now $\sin x < x, \forall\ x > 0$

$\therefore \quad f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}.$

22. (a) Given that P (3, 2, 6) is a point in space and Q is a point on line

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + m(-3\hat{i} + \hat{j} + 5\hat{k})$

or $\quad \frac{x-1}{-3} = \frac{y+1}{1} = \frac{z-2}{5} = \mu$

Let coordinates of Q be $(-3\mu + 1, \mu - 1, 5\mu + 2)$

$\therefore$ d.r's of $\overrightarrow{PQ} = -3\mu - 2,\ \mu - 3,\ 5\mu - 4$

As $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$

$\therefore \quad 1.(-3\mu - 2) - 4.(\mu - 3) + 3(5\mu - 4) = 0$

$\Rightarrow \quad 8\mu = 2$ or $\mu = \frac{1}{4}$

23. (c) $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors,

Let $\vec{a} \times \vec{b} = (\sin \alpha)\vec{n_1}$ and $\vec{c} \times \vec{d} = (\sin \beta)\vec{n_2}$

then $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = 1$

$\Rightarrow \quad (\sin\alpha)\vec{n_1}.(\sin\beta)\vec{n_2} = 1$

$\Rightarrow \quad \sin\alpha \sin\beta\ \vec{n_1}.\vec{n_2} = 1$

$\Rightarrow \quad \sin\alpha \sin\beta \cos\gamma = 1$

where γ is the angle between $\vec{n_1}$ and $\vec{n_2}$.

$\alpha = \frac{\pi}{2}, \beta = \frac{\pi}{2}$ and $\gamma = 0°$

Now $\gamma = 0° \Rightarrow \vec{a} \times \vec{b} \,||\, \vec{c} \times \vec{d}$

Let $\vec{a} \times \vec{b} = \lambda(\vec{c} \times \vec{d})$

$\Rightarrow \quad (\vec{a} \times \vec{b}).\vec{c} = \lambda(\vec{c} \times \vec{d}).\vec{c} = 0$

and $(\vec{a} \times \vec{b}).\vec{d} = \lambda(\vec{c} \times \vec{d}).\vec{d} = 0$

$\therefore \quad \vec{a}, \vec{b}, \vec{c}$ are coplanar and $\vec{a}, \vec{b}, \vec{d}$ are coplanar

$\Rightarrow \quad \vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar

Also $\alpha = 90°$

$\Rightarrow \quad \vec{a} \perp \vec{b}$ and $\beta = 90°$

$\Rightarrow \quad \vec{c} \perp \vec{d}$

But angle between $\vec{a}$ and $\vec{c}$ is $\pi/3$ $(\because\ \vec{a}.\vec{c} = \frac{1}{2})$

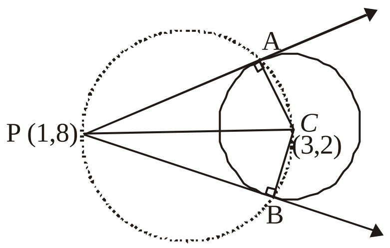
So, angle between $\vec{b}$ and $\vec{d}$ should also be $\pi/3$. Hence $\vec{b}$ and $\vec{d}$ are non parallel.

24. (c) We have to form 7 digit numbers, using the digits 1, 2 and 3 only, such that the sum of the digits in a number = 10.

This can be done by taking 2, 2, 2, 1, 1, 1, 1, or by taking 2, 3, 1, 1, 1, 1, 1.

$\therefore$ Number of ways $= \frac{7!}{3!4!} + \frac{7!}{5!} = 77.$

25. (b) Tangents PA and PB are drawn from the point P (1, 3) to circle $x^2 + y^2 - 6x - 4y - 11 = 0$ with centre C (3, 2)



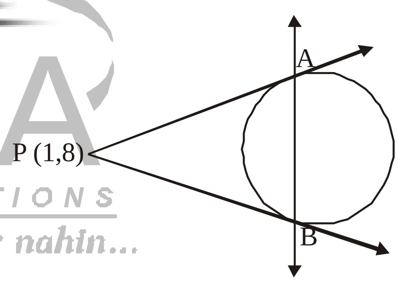
Clearly the circumcircle of ΔPAB will pass through C and as $\angle A = 90°$, PC must be a diameter of the circle.

$\therefore$ Equation of required circle is

$(x-1)(x-3) + (y-8)(y-2) = 0$

$\Rightarrow \quad x^2 + y^2 - 4x - 10y + 19 = 0$

Alternative :



Equation of chord of contact i.e. AB is

$x.1 + y.8 - 3(x+1) - 2(y+8) - 11 = 0$

or $\quad x - 3y + 15 = 0$

Now equation of circle passing through intersection point of circle $x^2 + y^2 - 6x - 4y - 11 = 0$ and of line $x - 3y + 15 = 0$ is given by $S + \lambda L = 0$

i.e. $(x^2 + y^2 - 6x - 4y - 11) + \lambda(x - 3y + 15) = 0$

As this circle passes through P (1, 8) also, we get

$(1 + 64 - 6 - 32 - 11) + \lambda\ (1 - 24 + 15) = 0$

$\Rightarrow \quad \lambda = 2$

$\therefore$ The required circle is

$(x^2 + y^2 - 6x - 4y - 11) + 2(x - 3y + 15) = 0$

or $x^2 + y^2 - 4x - 10y + 19 = 0$

26. (d) $z = \cos q + i \sin q$

$\Rightarrow \quad z^{2m-1} = (\cos q + i \sin q)^{2m-1}$

$= \cos(2m-1)\theta + i \sin(2m-1)\theta$

$\left[\begin{array}{l}\text{using De Moivre's theorem} \\ (\cos q + i\sin q)^n = \cos nq + i\sin nq\end{array}\right]$

$\therefore \quad \text{Im}(z^{2m-1}) = \sin(2m-1)q$

$\therefore \quad \sum_{m=1}^{15} \text{Im}(z^{2m-1}) = \sum_{m=1}^{15} \sin(2m-1)q$

$= \sin\theta + \sin 3\theta + \sin 5\theta + ... + \text{upto 15 terms}$

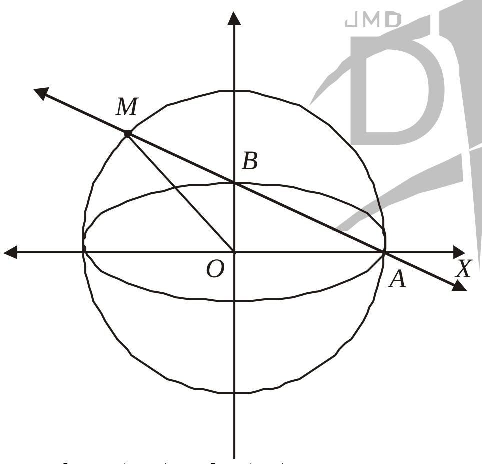
$$= \frac{\sin\left[15\left(\frac{2\theta}{2}\right)\right].\sin[\theta + 14\times\theta]}{\sin\theta}$$

$\left[\begin{array}{l}\text{Using } \sin a + \sin(a+b) + \sin(a+2b) + ...n \text{ terms} \\ = \dfrac{\sin(nb/2).\sin[a+(n-1)b/2]}{\sin(b/2)}\end{array}\right]$

$$= \frac{\sin 15\theta.\sin 15\theta}{\sin\theta} = \frac{\sin 30°.\sin 30°}{\sin 2°}$$

$$= \frac{1}{4\sin 2°}$$

27. **(d)** The given ellipse is $x^2 + 9y^2 = 9$ or $\frac{x^2}{3^2} + \frac{y^2}{1^2} = 1$



So, that $A(3,0)$ and $B(0,1)$

$\therefore$ Equation of AB is $\frac{x}{3} + \frac{y}{1} = 1$

or $x + 3y - 3 = 0$ (1)

Also auxillary circle of given ellipse is

$x^2 + y^2 = 9$ (2)

Solving equation (1) and (2), we get the point M where line AB meets the auxillary circle.

Putting $x = 3 - 3y$ from eq^n(1) in eq^n(2)

we get $(3-3y)^2 + y^2 = 9$

$\Rightarrow \quad 9 - 18y + 9y^2 + y^2 = 9$

$\Rightarrow \quad 10y^2 - 18y = 0$

$\Rightarrow \quad y = 0, \frac{9}{5}$

$\Rightarrow \quad x = 3, \frac{-12}{5}$

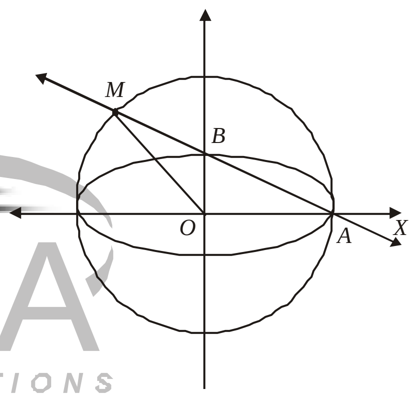
Clearly M $\left(\frac{-12}{5}, \frac{9}{5}\right)$

$$\therefore \quad \text{Area of } \Delta OAM = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & 0 & 1 \\ \frac{-12}{5} & \frac{9}{5} & 1 \end{vmatrix} = \frac{27}{10}$$

Alternative:

The given ellipse is $\frac{x^2}{3^2} + \frac{y^2}{1^2} = 1$, so that

$OA = 3, OB = 1$



Let $\angle OAM = \theta = \angle OMA$ $(\because OA = OM)$

In ΔOAB, $\tan\theta = \frac{1}{3}$

Also in ΔOAM, $\angle AOM = 180 - 2\theta$

So that $\sin\angle AOM = \sin(180 - 2\theta) = \sin 2\theta$

$$= \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\times\frac{1}{3}}{1+\frac{1}{9}} = \frac{3}{5}$$

$\therefore \quad \text{Ar}(\Delta OAM) = \frac{1}{2}\times OA^2 \times \sin\angle AOM$

$\left[\Delta = \frac{1}{2}r^2\sin\theta \text{ for isosceles triangle}\right]$

$$= \frac{1}{2}\times 3^2 \times \frac{3}{5} = \frac{27}{10}$$

28. **(a)** Given $z = x + iy$ where x and y are integer

Also $z\bar{z}^3 + \bar{z}z^3 = 350$

$\Rightarrow \quad |z|^2(\bar{z}^2 + z^2) = 350$

$\Rightarrow \quad (x^2 + y^2)(x^2 - y^2) = 175$

$\Rightarrow \quad (x^2+y^2)(x^2-y^2) = 25\times 7$ ...(i)

or $\quad (x^2+y^2)(x^2-y^2) = 35\times 5$ ...(ii)

$\because$ x and y are integers,

$\therefore \; x^2+y^2 = 25$ and $x^2-y^2 = 7$ [From eq (i)]

$\Rightarrow \; x^2 = 16$ and $y^2 = 9$

$\Rightarrow \; x = \pm 4$ and $y = \pm 3$

$\therefore$ Vertices of rectangle are
(4, 3) , (4, – 3), (– 4, – 3) , (– 4, 3).

So, area of rectangle = 8 × 6 = 48 sq. units

Now from eq. (ii)

or $\quad x^2 + y^2 = 35$ and $x^2 - y^2 = 5$

$\Rightarrow x^2 = 20$, which is not possible for any integral value of x

29. (a,c) Given that $L = \lim_{x\to 0} \dfrac{a - \sqrt{a^2 - x^2} - \dfrac{x^2}{4}}{x^4}, a > 0$

and L is finite.

Now $L = \lim_{x\to 0} \dfrac{\dfrac{x}{\sqrt{a^2-x^2}} - \dfrac{x}{2}}{4x^3}$

(Using L'Hospital's rule)

$= \lim_{x\to 0} \dfrac{\dfrac{1}{\sqrt{a^2-x^2}} - \dfrac{1}{2}}{4x^2}$

$\because$ L is finite, limiting value of numerator should be zero which is so when $\dfrac{1}{\sqrt{a^2}} - \dfrac{1}{2} = 0$

i.e. $a = 2$ $\quad (\because \; a > 0)$

Applying L 'Hospital's rule again, we get

$$L = \lim_{x\to 0} \frac{\dfrac{x}{(a^2-x^2)^{3/2}}}{8x}$$

$$= \lim_{x\to 0} \frac{1}{8(a^2-x^2)^{3/2}}$$

$$= \frac{1}{8\times a^3} = \frac{1}{8\times 8} \quad \text{(using } a = 2)$$

$$= \frac{1}{64}$$

30. (b, c) In ΔABC, given that

$$\cos B + \cos C = 4\sin^2 \frac{A}{2}$$

$$\Rightarrow \; 2\cos\frac{B+C}{2}\cos\frac{B-C}{2} - 4\sin^2\frac{A}{2} = 0$$

$$\Rightarrow \; 2\sin\frac{A}{2}\left[\cos\frac{B-C}{2} - 2\sin\frac{A}{2}\right] = 0$$

$$\Rightarrow \; \sin\frac{A}{2} = 0 \text{ or } \cos\frac{B-C}{2} - 2\cos\frac{B+C}{2} = 0$$

But in a triangle $\sin\dfrac{A}{2} \neq 0$

$$\therefore \quad \cos\frac{B-C}{2} - 2\cos\frac{B+C}{2} = 0$$

$$\Rightarrow \; \frac{\cos\left(\dfrac{B+C}{2}\right)}{\cos\left(\dfrac{B-C}{2}\right)} = \frac{1}{2}$$

Applying componendo and dividendo, we get

$$\frac{\cos\left(\dfrac{B+C}{2}\right) + \cos\left(\dfrac{B-C}{2}\right)}{\cos\left(\dfrac{B+C}{2}\right) - \cos\left(\dfrac{B-C}{2}\right)} = \frac{1+2}{1-2} = -3$$

$$\Rightarrow \; \frac{2\cos\dfrac{B}{2}\cos\dfrac{C}{2}}{-2\sin\dfrac{B}{2}\sin\dfrac{C}{2}} = -3$$

$$\Rightarrow \; \tan\frac{B}{2}\tan\frac{C}{2} = \frac{1}{3}$$

$$\Rightarrow \; \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{1}{3}$$

$$\Rightarrow \; \frac{s-a}{s} = \frac{1}{3} \text{ or } 2s = 3a$$

$\Rightarrow \; a + b + c = 3a \quad$ or $\quad b + c = 2a$

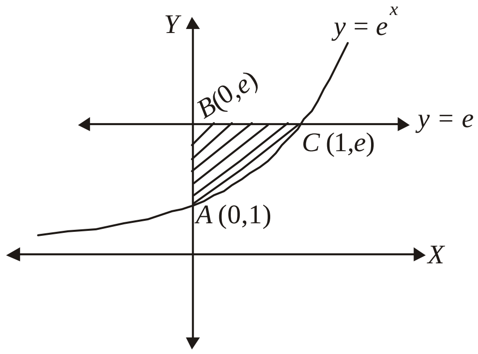
i.e. $AC + AB$ = constant

($\because$ Base $BC = a$ is given to be constant)

$\Rightarrow$ A moves on an ellipse.

31. (b, c, d)

The area bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is as shown in the graph.



Required area

$$= \int_0^1 (e - e^x)dx = [ex]_0^1 - \int_0^1 e^x dx$$

$$= e - \int_0^1 e^x dx. = 1$$

Also required area

$$= \int_0^e x\,dy \qquad (\text{where } e^x = y \;\Rightarrow\; x = \ln y)$$

$$= \int_1^e \ln y\,dy$$

$$= \int_1^e \ln(e+1-y)\,dy \quad \left[\begin{array}{l}\text{Using the property} \\ \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\end{array}\right]$$

32. (a, b)

Given that

$$\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$$

$$\Rightarrow \quad 3\sin^4 x + 2\cos^4 x = \frac{6}{5}$$

$$\Rightarrow \quad \sin^4 x + 2[\sin^4 x + \cos^4 x] = \frac{6}{5}$$

$$\Rightarrow \quad \sin^4 x + 2[1 - 2\sin^2 x\cos^2 x] = \frac{6}{5}$$

$$\Rightarrow \quad \sin^4 x + 2 - 4\sin^2 x(1 - \sin^2 x) = \frac{6}{5}$$

$$\Rightarrow \quad 5\sin^4 x - 4\sin^2 x + 2 - \frac{6}{5} = 0$$

$$\Rightarrow \quad 25\sin^4 x - 20\sin^2 x + 4 = 0$$

$$\Rightarrow \quad (5\sin^2 x - 2)^2 = 0$$

$$\Rightarrow \quad \sin^2 x = \frac{2}{5}$$

$$\Rightarrow \quad \cos^2 x = \frac{3}{5} \quad \text{and} \quad \tan^2 x = \frac{2}{3}$$

Also $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{625} + \frac{3}{625} = \frac{5}{625} = \frac{1}{125}$

33. (a) $\because$ Each element of set A is 3 × 3 symmetric matrix with five of its entries as 1 and four of its entries as 0, we can keep in diagonal either 2 zero and one 1 or no zero and three 1 so that the left over zeros and one's are even in number.

Hence taking 2 zeros and one 1 in diagonal the possible cases are $\frac{3!}{2!} \times \frac{3!}{2!} = 9$

and taking 3 ones in diagonal the possible cases are

$1 \times \frac{3!}{2!} = 3$

$\therefore$ Total elements A can have = 9 + 3 = 12

34. (b) The given system will have unique solution if $|A| \neq 0$ which is so for the matrices.

$$\begin{bmatrix}1&0&1\\0&0&1\\1&1&0\end{bmatrix} \text{ and } \begin{bmatrix}1&1&0\\1&0&1\\0&1&0\end{bmatrix};$$

$$\begin{bmatrix}0&0&1\\0&1&1\\1&1&0\end{bmatrix} \text{ and } \begin{bmatrix}0&1&1\\1&1&0\\1&0&0\end{bmatrix};$$

$$\begin{bmatrix}0&1&1\\1&0&0\\1&0&1\end{bmatrix} \text{ and } \begin{bmatrix}0&1&0\\1&0&1\\0&1&1\end{bmatrix}$$

which are 6 in number.

35. (b) For the given system to be inconsistent $|A| = 0$. The matrices for which $|A| = 0$ are

$$\underset{\text{(i)}}{\begin{bmatrix}1&0&0\\0&1&1\\0&1&1\end{bmatrix}}, \underset{\text{(ii)}}{\begin{bmatrix}1&0&1\\0&1&0\\1&0&1\end{bmatrix}}, \underset{\text{(iii)}}{\begin{bmatrix}1&1&0\\1&1&0\\0&0&1\end{bmatrix}}$$

and

$$\underset{\text{(iv)}}{\begin{bmatrix}1&1&1\\1&0&0\\1&0&0\end{bmatrix}}, \underset{\text{(v)}}{\begin{bmatrix}0&1&0\\1&1&1\\0&1&0\end{bmatrix}}, \underset{\text{(vi)}}{\begin{bmatrix}0&0&1\\0&0&1\\1&1&1\end{bmatrix}}$$

On solving $A\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$

We find for A = (i)

By Cramer's rule $D_1 = 0 = D_2 = D_3$

$\therefore$ infinite many solution

For A = (ii)

By Cramer's rule $D_1 \neq 0$

$\Rightarrow$ no solution i.e. inconsistent.

Similarly we find the system as inconsistent in cases (iii), (v) and (vi).

Hence for four cases system is inconsistent.

36. (a) $P(X = 3)$ = (probability of not a six in first chance) × (probability of not a six in second chance) × (probability of a six in third chance)

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

37. (b) $P(X \geq 3) = 1 - (X < 3)$

$$= 1 - [P(X = 1) + P(X = 2)]$$

$$= 1 - \left[\frac{1}{6} + \frac{5}{6} \times \frac{1}{6}\right] = 1 - \frac{11}{36} = \frac{25}{36}$$

Alternative:

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5) + \ldots\infty$$

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots\infty$$

$$= \frac{25}{216}\left[1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \ldots\infty\right]$$

$$= \frac{25}{216} \times \frac{1}{1 - \frac{5}{6}} = \frac{25}{216} \times \frac{6}{1} = \frac{25}{36}$$

38. (d) Let us define the events

$A \equiv X \geq 6$ and $B \equiv X > 3$

so that $A \cap B \equiv X \geq 6 \equiv A$

Now $P(A) = \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \times \frac{1}{6} + \ldots .\infty$

$= \left(\frac{5}{6}\right)^5 \times \frac{1}{6}\left[1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \ldots .\infty\right]$

$= \left(\frac{5}{6}\right)^5 \times \frac{1}{6} \times \frac{1}{1 - \frac{5}{6}} = \left(\frac{5}{6}\right)^5$

and $P(B) = \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \ldots .\infty = \left(\frac{5}{6}\right)^3$

$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$

39. ***A* *p,q,s;* *B* *p,t;* *C* *p,q,r,t;* *D* *s***

(A) $(x-3)^2 y' + y = 0$

$\Rightarrow \quad (x-3)^2 \frac{dy}{dx} = -y$

$\Rightarrow \quad \int -\frac{1}{y} dy = \int \frac{1}{(x-3)^2} dx$

or $\quad \log|y| = \frac{1}{x-3} + \log c,\ x \neq 3$

$\Rightarrow \quad \log\left(\frac{y}{c}\right) = \frac{1}{x-3},\ x \neq 3$

$\Rightarrow \quad \frac{y}{c} = e^{\frac{1}{x-3}}$ or $y = ce^{\frac{1}{x-3}},\ x \neq 3$

$\therefore$ The solution set is $(-\infty, \infty) - \{3\}$

This set contains the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$, and $\left(0, \frac{\pi}{8}\right)$.

$\therefore \quad$ ***(A)*** $\rightarrow$ ***p, q, s***

(B) $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$

Let $(x-3) = t \Rightarrow dx = dt$

Also when $x \rightarrow 1,\ t \rightarrow -2$

and when $x \rightarrow 5,\ t \rightarrow 2$

$\therefore$ Integral becomes

$\int_{-2}^{2} (t+2)(t+1)t(t-1)(t-2)dt$

$= \int_{-2}^{2} t(t^2-1)(t^2-4)dt = 0$

as integrand is an odd function.

O is contained by $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $(-\pi, \pi)$

$\therefore \quad$ ***(B)*** $\rightarrow$ ***p, t.***

(C) Let $f(x) = \cos^2 x + \sin x$

$\Rightarrow \quad f'(x) = -2\sin x \cos x + \cos x$

For critical point $f'(x) = 0$

$\Rightarrow \quad \sin x = \frac{1}{2}$ or $\cos x = 0$

$\Rightarrow \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$, or $x = \frac{\pi}{2}, -\frac{\pi}{2}$

Now $f''(x) = -2\cos 2x - \sin x$

$f''(x)\big|_{x=\pi/6} = -ve$

$f''(x)\big|_{x=5\pi/6} = -ve$

$f''(x)\big|_{x=\pi/2} = +ve$ and $f''(x)\big|_{x=-\pi/2} = +ve$

$\therefore \quad \frac{\pi}{6}$ and $\frac{5\pi}{6}$ are the points of local maxima.

Clearly all the intervals given in column II except $\left(0, \frac{\pi}{8}\right)$ contain at least one point of local maxima.

$\therefore \quad$ ***(C)*** ® ***p, q, r, t***

(D) Let $f(x) = \tan^{-1}(\sin x + \cos x)$

$= \tan^{-1}\left[\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right]$

$f'(x) = \frac{1}{1 + 2\sin^2\left(x + \frac{\pi}{4}\right)} . \sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$

For $f(x)$ to be an increasing function,

$f'(x) > 0$

$\Rightarrow \quad \cos\left(x + \frac{\pi}{4}\right) > 0 \Rightarrow -\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$

$\Rightarrow \quad -\frac{3\pi}{4} < x < \frac{\pi}{4}$

Clearly only $\left(0, \frac{\pi}{8}\right) \subset \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$

$\therefore \quad$ ***(D)*** $\rightarrow$ ***s.***

40. *A* → *p*; *B* → *s*, *t*; *C* → *r*; *D* → *q*, *s*

(p) As the line $hx+ky=1$, touches the circle $x^2+y^2=4$

$\therefore$ Length of perpendicular from centre (0 , 0) of circle to line = radius of the circle

$$\Rightarrow \frac{1}{\sqrt{h^2+k^2}}=2 \Rightarrow h^2+k^2=\frac{1}{4}$$

$\therefore$ Locus of (*h*, *k*) is $x^2+y^2=\frac{1}{4}$, which is a circle.

(q) We know that if $|z-z_1|-|z-z_2|=k$

where $|k|<|z_1-z_2|$

then *z* traces a hyperbola.

Here $|z+2|-|z-2|=3$

$\therefore$ Locus of *z* is a hyperbola.

(r) We have $x=\sqrt{3}\left(\frac{1-t^2}{1+t^2}\right),\ y=\frac{2t}{1+t^2}$

$$\Rightarrow \frac{x}{\sqrt{3}}=\frac{1-t^2}{1+t^2} \text{ and } y=\frac{2t}{1+t^2}$$

On squaring and adding, we get

$$\frac{x^2}{3}+y^2=\frac{(1-t^2)^2+4t^2}{(1+t^2)^2}=1$$

or $\frac{x^2}{3}+\frac{y^2}{1}=1$

which is the equation of an ellipse.

(s) We know eccentricity for a parabola = 1
for an ellipse < 1
for a hyperbola > 1

$\therefore$ The conics whose eccentricity lies in $1 \le x < \infty$ are parabola and hyperbola.

(t) Let $z=x+iy$ then

$$\text{Re }[(x+1)+iy]^2=x^2+y^2+1$$

$$\Rightarrow (x+1)^2-y^2=x^2+y^2+1$$

$\Rightarrow y^2=x$, which is a parabola.

PHYSICS

41. **(c)** Consider the activitiy *A* to *B*.

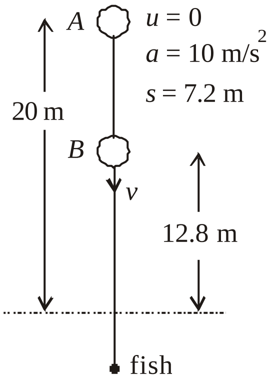
Applying $v^2-u^2=2as$

$$v^2-0^2=2\times 10\times 7.2$$

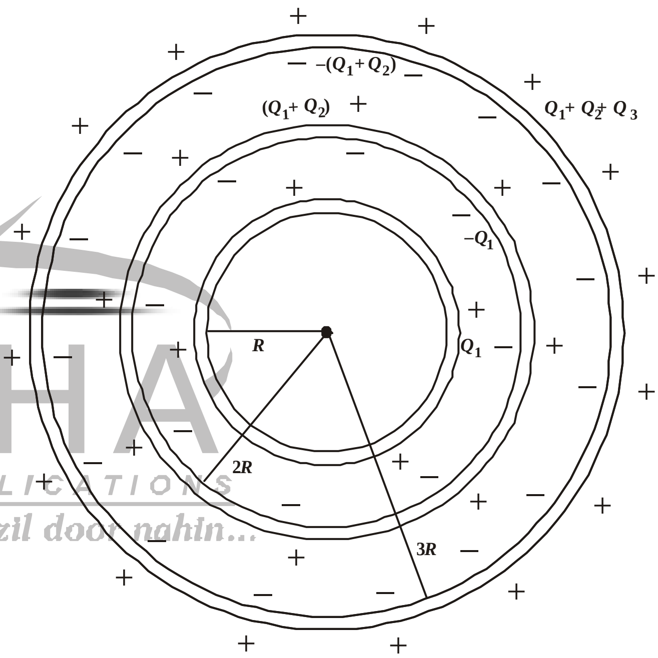
$$\Rightarrow v=12\text{m/s}$$

The velocity of ball as perceived by fish is

$$v'=\mu_w\times v=\frac{4}{3}\times 12=16\text{m/s}$$



42. **(b)** The charges on the surfaces of the metallic spheres are shown in the diagram. It is given that the surface charge densities on the outer surfaces of the shells are equal. Therefore



$$\frac{Q_1}{4\pi R^2}=\frac{Q_1+Q_2}{4\pi(2R)^2}=\frac{Q_1+Q_2+Q_3}{4\pi(3R)^2}=x(\text{say})$$

$\therefore$ $Q_1=4\pi R^2 x$

$Q_1+Q_2=4\pi(2R)^2x=4[4\pi R^2x]$

$\Rightarrow Q_2=4[4\pi R^2x]-Q_1$

$=4[4\pi R^2x]-4\pi R^2x=3[4\pi R^2x]$

Also $Q_1+Q_2+Q_3=4\pi(3R)^2x=9[4\pi R^2x]$

$\therefore Q_3=9[4\pi R^2x]-Q_1-Q_2=9[4\pi R^2x]-[4\pi R^2x]$

$-3[4\pi R^2x]=5[4\pi R^2x]$

$\Rightarrow Q_1:Q_2:Q_3=1:3:5$

43. **(d)** The magnetic field is increasing in the downward direction. Therefore, according to Lenz's law the current I_1 will flow in the direction **ab** and I_2 in the direction **dc**.

44. (d) From the graph it is clear that the amplitude is 1 cm and the time period is 8 second. Therefore, the equation for the S.H.M. is

$$x = a\sin\left(\frac{2\pi}{T}\right)\times t \;= 1\sin\left(\frac{2\pi}{8}\right)t$$

$$x = \sin\frac{\pi}{4}t$$

The velocity (v) of the particle at any instant of time 't' is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[\sin\left(\frac{\pi}{4}\right)t\right] = \frac{\pi}{4}\cos\left(\frac{\pi}{4}\right)t$$

The acceleration of the particle is

$$\frac{d^2x}{dt^2} = -\left(\frac{\pi}{4}\right)^2 \sin\left(\frac{\pi}{4}\right)t$$

At $t = \frac{4}{3}$s we get

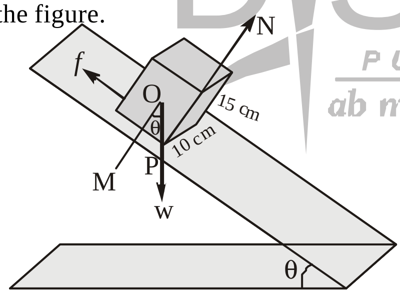
$$\frac{d^2x}{dt^2} = -\left(\frac{\pi}{4}\right)^2 \sin\frac{\pi}{4}\times\frac{4}{3} = \frac{-\pi^2}{16}\sin\frac{\pi}{3}$$

$$= \frac{-\sqrt{3}\pi^2}{32}\text{cm/s}^2$$

45. (b) For the block to slide, the angle of inclination should be equal to the angle of repose, i.e.,

$\tan^{-1}\mu \;= \tan^{-1}\sqrt{3} = 60°$. Therefore, option (a) is wrong.

For the block to topple, the condition of the block will be as shown in the figure.

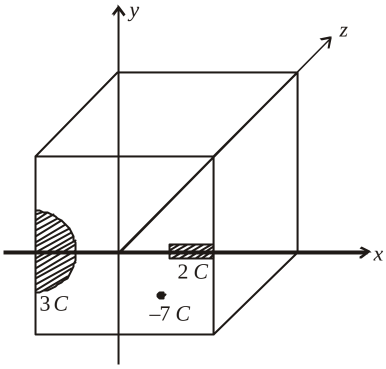


In ΔPOM,

$$q = \tan\theta = \frac{PM}{OM} = \frac{5\text{cm}}{7.5\text{cm}} = \frac{2}{3}$$

For this, $\theta < 60°$. From this we can conclude that the block will topple at lesser angle of inclination. Thus the block will remain at rest on the plane up to a certain anlgle θ and then it will topple.

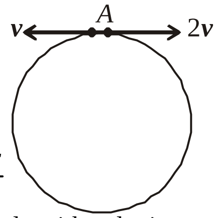
46. (a)



From the figure it is clear that the charge enclosed in the cubical surface is $3C + 2C - 7C = -2C$. Therefore the electric flux through the cube is

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{-2C}{\varepsilon_0}$$

47. (c) Let the radius of the circle be r. Then the two distance travelled by the two particles before first collision is $2\pi r$. Therefore



$$2v\times t + v\times t = 2\pi r$$

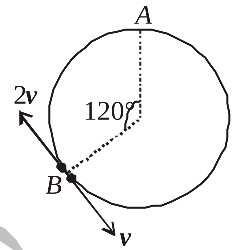
where t is the time taken for first collision to occur.

$\therefore \quad t = \frac{2\pi r}{3v}$

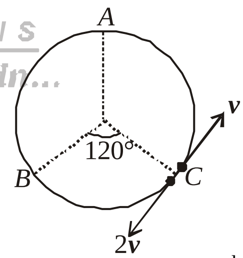
$\therefore$ Distance travelled by particle with velocity

$$v = v\times\frac{2\pi r}{3v} = \frac{2\pi r}{3}.$$

Therefore, the collision occurs at B.

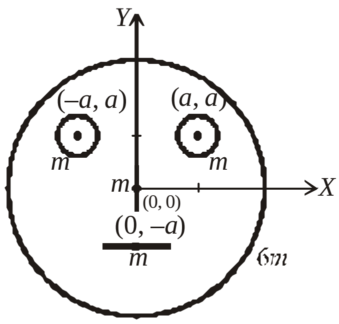


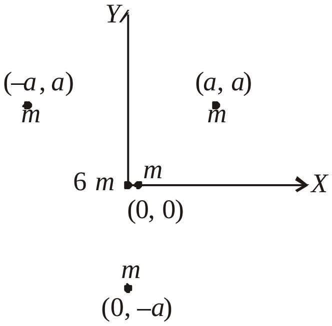
As the collision is elastic and the particles have equal masses, the velocities will interchange as shown in the figure. According to the same reasoning as above, the 2nd collision will take place at C and the velocities will again interchange.



With the same reasoning the 3rd collision will occur at A. Thus, there will be two elastic collisions before the particles again reach at A.

48. (a) The system is made up of five bodies (three circles and two straight lines) of uniform mass distribution. Therefore, we assume the system to be made up of five point masses where the mass of each body is considered at its geometrical centre.





The y-coordinate of the centre of mass is

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4 + m_5y_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$\therefore \quad y_{cm} = \frac{6m\times 0 + m\times 0 + m\times a + m\times a + m(-a)}{6m+m+m+m+m}$$

$$= \frac{ma}{10m} = \frac{a}{10}$$

49. (c, d) Given $f = -24$ cm

Applying $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

For (66, 33)

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-24}+\frac{1}{66}=\frac{-66+24}{24\times 66}=\frac{-42}{24\times 66}$$

$$\Rightarrow \quad v = -\frac{24\times 66}{42} = -37.7$$

But the value of v = 33. The absolute error is 37.7 – 33 = 4.7 cm which is greater than 0.2 cm. Therefore, it is a wrong reading.

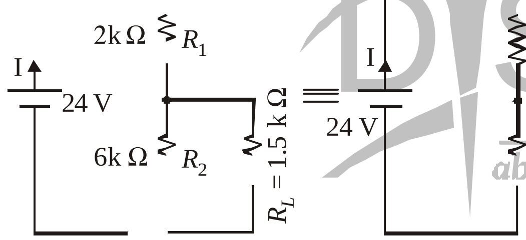
For (78, 39), when u = 78 then

$$\frac{1}{v}+\frac{1}{-78}=\frac{1}{-24}$$

$$\Rightarrow \quad v = -34.67$$

The absolute error is 39 – 34.67 = 4.33 which is greater than 0.2 cm.

50. (a, d)



$$R_p = \frac{R_2\times R_L}{R_2+R_L}=\frac{6\times 1.5}{6+1.5}=\frac{9}{7.5}k\Omega$$

$$\therefore \quad I = \frac{24}{\frac{9}{7.5}+2}\text{mA} = \frac{24\times 7.5}{24} = 7.5\text{mA}$$

$\Rightarrow$ option (a) is correct.

The potential difference across

R_L = potential difference across R_p

$$= (7.5\text{mA})\left(\frac{9}{7.5}\text{k}\Omega\right) = 9V$$

$\Rightarrow$ option (b) is incorrect.

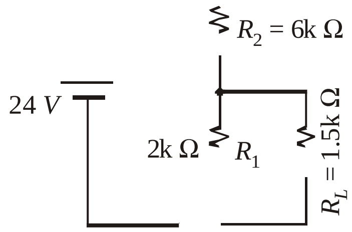
$$\text{Now,} \quad \frac{\text{Power dissipation across } R_1}{\text{Power dissipation across } R_2} = \frac{\frac{(15)^2}{2\times 10^3}}{\frac{(9)^2}{6\times 10^3}}$$

$$= \frac{15\times 15}{2}\times\frac{6}{9\times 9} = 8.33$$

$\Rightarrow$ option (c) is incorrect.

The magnitude of power dissipated across R_L is $\frac{(9)^2}{1.5\times 10^3}$ watt.

Now, when R_1 and R_2 are interchanged, the equivalent resistance between R_1 and $R_L = \frac{2\times 1.5}{2+1.5}=\frac{3}{3.5}k\Omega$



$\therefore$ Potential drop across this equivalent resistance

$$= \frac{\frac{3\times 10^3}{3.5}}{\left(\frac{3}{3.5}+6\right)\times 10^3}\times 24 = \frac{3}{24}\times 24 = 3$$

$\therefore$ Potential difference across

$$R_L = \frac{3^2}{1.5\times 10^3} = \frac{1}{9}\left[\frac{9^2}{1.5\times 10^3}\right]$$

$\therefore$ The magnitude of the power dissipation in R_L will decrease by a factor 9 if R_1 and R_2 are interchanged. (d) is the correct option.

51. (a) $\sum \vec{F}_{ext} = \frac{d\,\vec{p}_{\text{system}}}{dt}$

Given $\sum \vec{F}_{ext} = 0 \Rightarrow \vec{p}_{\text{system}}$ = Constant

Due to internal forces acting in the system, the kinetic and potential energy may change with time.

Also zero external force may create a torque if the line of action of forces are along different directions. Thus the torque will change the angular momentum of the system.

52. (b, d) $C_p - C_v = R$ for all gases

For monoatomic gas :

$$C_v = \frac{3}{2}R \;;\; C_p = \frac{5}{2}R \;;\; \gamma = \frac{Cp}{C_v} = \frac{5}{3}$$

$$C_p.C_v = \frac{15}{4} \;;\; C_p + C_v = 4$$

For diatomic gas :

$$C_v = \frac{5}{2}R \;;\; C_p = \frac{7}{2}R \;;\; \gamma = \frac{C_p}{C_v} = \frac{7}{5} \text{ and}$$

$$C_p.C_v = \frac{35}{4} \;;\; C_p + C_v = 6$$

53. (d) The collection of $^2_1$H nuclei and electron is known as plasma which is formed due to high temperature inside the reactor core.

54. **(a)** Applying conservation of mechanical energy, we get,
Loss of kinetic energy of two deuteron nuclei
= Gain in their potential energy.

$$2\times 1.5kT = \frac{1}{4\pi\varepsilon_0}\frac{e\times e}{r}$$

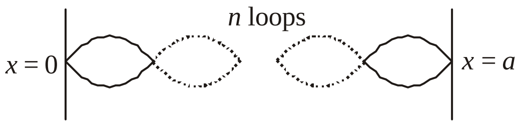
$$\Rightarrow 2\times 1.5\times\left(8.6\times 10^{-5}\,\frac{\text{eV}}{\text{K}}\right)\times T = \frac{(1.44\times 10^{-9}\,\text{eVm})}{4\times 10^{-15}\,\text{m}}$$

$$\Rightarrow \quad T = \frac{1.44\times 10^{-9}}{2\times 1.5\times 8.6\times 10^{-5}\times 4\times 10^{-15}}$$

$= 0.0139 \times 10^{11} = 1.4 \times 10^{9}$ K

55. **(b)** For the reading given in option (b), we get, $nt_o > 5\times 10^{14}$ which is the Lawson criterion for a reactor to work successfully.

56. **(a)** $\lambda = \dfrac{h}{p}$ and $E = \dfrac{p^2}{2m}$



$$\Rightarrow \quad E = \frac{h^2}{2m\lambda^2}$$

The length in which the particle is restricted to move is a. This length is a multiple of $\dfrac{\lambda}{2}$.

Now, $n\dfrac{\lambda}{2} = a \;\Rightarrow\; \lambda = \dfrac{2a}{n}$

$$\Rightarrow \quad E = \frac{h^2 n^2}{2m\times 4a^2} = \frac{n^2h^2}{8ma^2}$$

$\Rightarrow \quad E \propto a^{-2}$ for a particular value of n.

57. **(b)** For ground state $n = 1$,
Given $m = 1.0 \times 10^{-30}$ kg, $a = 6.6 \times 10^{-9}$ m

$$\therefore \quad E = \frac{1^2\times(6.6\times 10^{-34})^2}{8\times 1\times 10^{-30}\times(6.6\times 10^{-9})^2}\,\text{J} = \frac{10^{-68}}{8\times 10^{-48}}\,\text{J}$$

$$= \frac{10^{-20}}{8}\,\text{J}$$

$$= \frac{10^{-20}}{8\times 1.6\times 10^{-19}}\,\text{eV} = \frac{100}{8\times 1.6}\,\text{meV} \simeq 8\,\text{meV}$$

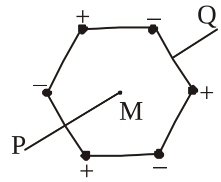
58. **(d)** $\lambda = \dfrac{h}{p} \;\Rightarrow\; \lambda = \dfrac{h}{mv} \;\Rightarrow\; mv = \dfrac{h}{\lambda}$

But $\dfrac{n\lambda}{2} = a \;\Rightarrow\; \lambda = \dfrac{2a}{n}$

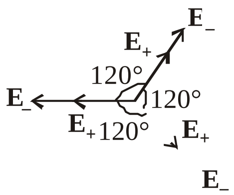
$\therefore \quad mv = \dfrac{nh}{2a} \;\Rightarrow\; v = \dfrac{nh}{2am} \;\Rightarrow\; v \propto n$

59. $[A \rightarrow (p, r, s)], [B \rightarrow (r, s)], [C \rightarrow (p, q, t)], [D \rightarrow (r, s)]$

(p)

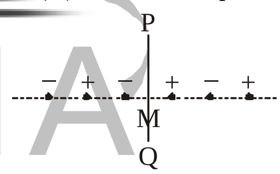


The electric field at M due to the charges at the corners of regular hexagon is as shown



Here $|\mathbf{E}_+| = |\mathbf{E}_-|$. The symmetry of the situation shows that E = 0 at M.
Therefore, (A) is the correct option.
The electric potential due to all the charges at M is zero.
Therefore, (B) is incorrect option.
When the system of charges is rotated about line PM, the net current will be zero.
Therefore, the magnetic field at M is zero.
Hence, (C) is the correct option.
When magnetic field is zero, then $\mu = 0$
$\therefore$ (D) is incorrect option.

(q)

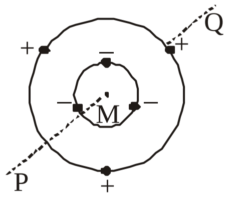


The electric field due to the inner most positive and negative charges at M is $E_1 = 2\left[k\dfrac{q}{r^2}\right]$ towards left.

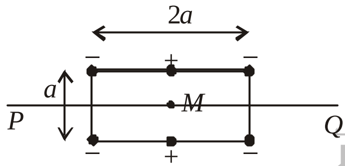
The electic field due to the next positive and negative charges at M is $E_2 = 2\left[k\dfrac{q}{(3r)^2}\right]$ towards right. The electric field due to the next positive and negative charges at M is $E_3 = 2\left[k\dfrac{q}{(5r)^2}\right]$ towards left. Similarly, it will go on. Clearly the vector sum of these electric fields is not zero.
(A) is incorrect option.
The electric potential due to the charges at M

$$= k\left[\frac{q}{r} - \frac{q}{r} + \frac{q}{3r} - \frac{q}{3r} + \frac{q}{5r} - \frac{q}{5r} + ...\right] = 0$$

(B) is incorrect option.
The net current due to the innermost positive and negative charges is zero. Similarly, the net current due to other charges in pair is zero. Therefore the magnetic field at M is zero. Also the magnetic moment is zero.
(C) is the correct option
(D) is incorrect option.

(r) 

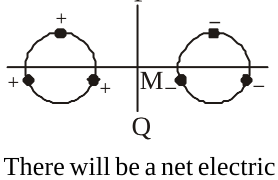
The net electric field due to negative charges in the inner circle is zero. Similarly, the net electric field due to positive charges in the outer circle is zero.
(A) is the correct option.
The electric potential due to negative charges at M is different from the electric potential due to positive charges at M. Therefore, the electric potential at M is not equal to zero.
(B) is the correct option.
When the system of charges rotate, we get a current I_1 due to negative charges and another current I due to positive charges. The magnitude of the magnetic field at M due to the currents is different. Therefore, $B \neq 0$ and $\mu \neq 0$
(C) is incorrect option.
(D) is the correct option.

(s) 

The electric field at M due to all the charges is zero because the electric field due to different charges cancel out in pairs.
(A) is the correct option.
The potential at M due to the charges is

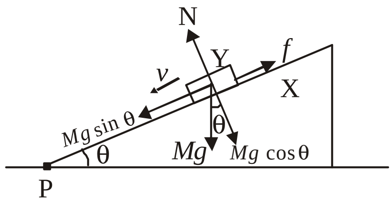
$$V = k\left[\frac{+q}{a/2} + \frac{q}{a/2} - 4\left(\frac{q}{\frac{\sqrt{5}a}{4}}\right)\right] \neq 0$$

(B) is the correct option.
When the whole system is set into rotation with a constant angular velocity about the line PQ, we get three loops in which current is flowing.
The magnetic field due to these currents produce a resultant magneic field at M which is not equal to zero. Therefore, a net magnetic dipole moment will be produced.
(C) is an incorrect option.
(D) is correct option.

(t) 

There will be a net electric field due to the arrangement of charges at M towards the right side.
(A) is an incorrect option.
The electic potential at M will cancel out in pairs by positive and negative charges, due to symmetrical arrangement of charges.
(B) is an incorrect option.
When the system of charges rotates about PQ, the net current is zero due to symmetrical arrangement of charges. Therefore, $B = 0$ and $\mu = 0$
(C) is the correct option.
(D) is the incorrect option.

60. $[A \to (p, t)]$; $[B \to (q, s, t)]$; $[C \to (p, r, t)]$ $[D \to (q)]$

(p) 

As the velocity is constant,

$$f = Mg\sin\theta \quad \text{..... (i)}$$

But $f = \mu N = \mu Mg\cos\theta$ ..... (ii)

From (i) and (ii),

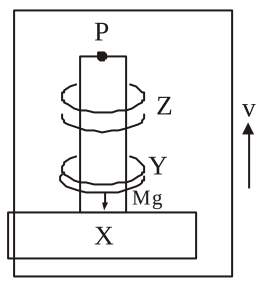
$$\mu Mg\cos\theta = Mg\sin\theta \Rightarrow \mu = \tan\theta$$

The force by X on Y is the resultant of f and N and is equal to

$$\sqrt{f^2 + N^2} = \sqrt{\mu^2 N^2 + N^2} = (\sqrt{\mu^2 + 1})N$$

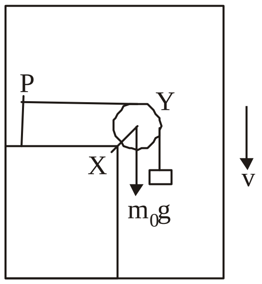
$$= (\sqrt{\tan^2\theta + 1})\ Mg\cos\theta = \sec\theta Mg\cos\theta = Mg$$

= weight of Y.
Therefore, option (A) is correct.
Now, due to the presence of frictional force between Y and X, the mechanical energy of the system $(X + Y)$ decreases continously as Y slides down.
Therefore, option (C) is correct.

(q) 

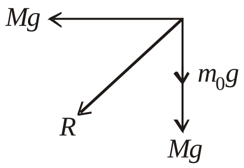
As the lift moves up, X also moves up and therefore the gravitational energy of X is continously increasing.
$\therefore$ Option (B) is correct.
The torque of the weight of Y about P is zero as the perpendicular distance of the line of action of force from the point P is zero.
$\therefore$ Option (D) is correct.
The force exerted by X on Y will be equal to $Mg + Mg = 2Mg$ where Mg is wt. of Y and Mg is the force on Y due to Z.
Option (A) is incorrect.

(r)



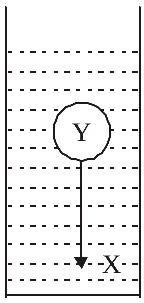
In this case the force exerted by X on Y is same as the force exerted by Y on X. The force on X due to Y is

$$R = \sqrt{(Mg)^2 + [(m_0 + M)g]^2} \neq Mg$$



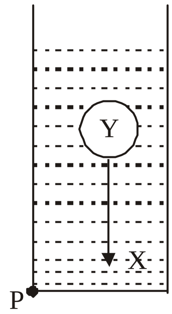
Therefore, option (A) is incorrect.
The mechanical energy of the system (X + Y) is continously decreasing as the system is coming down and its potential energy is decreasing, the kinetic energy remaining the same.
Therefore, option (C) is correct and (B) is incorrect.
The torque of the weight of Y about P is not zero.

(s)

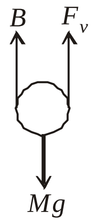


The force on Y by X is equal to the wt. of liquid displaced which cannot be equal to Mg as the density of Y is greater than density of X (as Y is sinking)
Therefore, option (A) is incorrect.
The gravitational potential energy of X increases continously because as Y moves down, the centre of mass of X moves up.
Therefore, option (B) is correct.

(t)



Sphere Y is moving with terminal velocity. Therefore, the net force on Y is zero, i.e.,



$$Mg = B + F_v$$

where B = buoyant force and F_v = viscous force.
$B + F_v$ are exerted by X on Y.
Therefore, option (A) is correct.
The gravitational potential energy of X is continously increasing because as Y moves down, the centre of mass of X moves up.
Option (B) is correct.
The mechanical energy of the system (X + Y) is continously decreasing to overcome the viscous forces.
Option (C) is correct.