DCE 2008 SOLVED PAPER

[Solutions]

THIS PAPER HAS BEEN TAKEN FROM "Disha's DCE Solved Papers"

PHYSICS

1. (b) The magnetic flux linked with the primary coil is given by
$\phi = \phi_0 + 4t$
So, voltage across primary
$$V_p = \frac{d\phi}{dt} = \frac{d}{dt}(\phi_0 + 4t) = \frac{d\phi_0}{dt} + \frac{4dt}{dt} = 4V$$
Using the relation,
$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \text{ or } V_s = V_p \frac{N_s}{N_p} = 4\left(\frac{1500}{50}\right)$$
$$= 120\,V$$

2. (c) Let displacement of particle executing SHM is $y = a \sin \omega t$
As particle travels half of the amplitude from the equilibrium position, so $y = \frac{a}{2}$
Therefore,
$$\frac{a}{2} = a\sin \omega t \text{ or } \sin \omega t = \frac{1}{2} = \sin\frac{\pi}{6}$$
$$\text{or } \omega t = \frac{\pi}{6} \text{ or } t = \frac{\pi}{6\omega}$$
$$\text{or } t = \frac{\pi}{6\left(\frac{2\pi}{T}\right)} \qquad \left(\text{as } \omega = \frac{2\pi}{T}\right)$$
$$\text{or } t = \frac{T}{12}$$
Hence, the particle travels half of the amplitude from the equilibrium in $\frac{T}{12}$ s.

3. (a) When a pure semiconductor like silicon is doped with a trivalent atom of aluminium (valency = 3), a p-type semiconductor is obtained in which no. of holes is greater than no. of electrons. The acceptor energy level lies just above the valence band.

4. (d) Number of nuclei remained after time t is given by $N = N_0 e^{-\lambda t}$
where N_0 is initial number of nuclei.
Thus, for two substances A and B, member of nuclei will be
$$N_1 = N_0 e^{-5\lambda t} \quad \text{... (i)}$$
$$\text{and } N_2 = N_0 e^{-\lambda t} \quad \text{... (ii)}$$
Dividing eq. (i) by eq. (ii), we obtain
$$\frac{N_1}{N_2} = \frac{e^{-5\lambda t}}{e^{-\lambda t}}$$
$$\frac{N_1}{N_2} = e^{(-5\lambda + \lambda)t} = e^{-4\lambda t} = \frac{1}{e^{4\lambda t}}$$
But given that, $\frac{N_1}{N_2} = \left(\frac{1}{e}\right)^2 = \frac{1}{e^2}$
Hence, $\frac{1}{e^2} = \frac{1}{e^{4\lambda t}}$
Comparing the powers, we get $2 = 4\lambda t$
$$\text{or } t = \frac{2}{4\lambda} = \frac{1}{2\lambda}$$

5. (a) The current due to revolving charge is given by $I = \frac{q}{T} = \frac{q}{\frac{2\pi}{\omega}} = \frac{q\omega}{2\pi}$
And, $\omega = \frac{v}{R}$ where v is the uniform speed and R is the radius of circle.
Thus, $I = \frac{qv}{2\pi R}$
Hence, magnetic moment
$$\mu = IA = \frac{qv}{2\pi R} \times \pi R^2 = \frac{1}{2} qvR$$

6. (a) Using the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$
$f = 10$ cm, $u = -8$ cm
Thus, $\frac{1}{10} = \frac{1}{v} - \frac{1}{-8}$
$$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{8} \Rightarrow \frac{1}{v} = \frac{8-10}{80}$$
$$\therefore v = \frac{80}{-2} = -40\text{cm}$$
Hence, magnification produced by the lens
$$m = \frac{v}{u} = \frac{-40}{-8} = 5$$

7. (a) Using, $eV_0 = h\nu_{max}$ or $eV_0 = \frac{hc}{\lambda_{min}}$

$\therefore V_0 = \frac{hc}{e\lambda_{min}}$

or $V_0 = \frac{12400 \times 10^{-10}}{10^{-11}} = 124\,kV$

Hence, accelerating voltage for electrons in X-ray machine should be less than 124kV.

8. (b) Photo electrons are emitted, when the frequency of incident light is greater than the minimum cut-off frequency (threshold frequency).

9. (b) Using the formula

$\frac{1}{f} = (n_{g_a} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = 0.5\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ... (i)

When the lens is immersed in a liquid of refractive index 1.25

$\frac{1}{f'} = \left(\frac{n_{g_a}}{n_{g_\ell}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= \left(\frac{1.5}{1.25} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\frac{1}{f'} = \left(\frac{0.25}{1.25}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ... (ii)

Dividing (i) by (ii)

$\frac{f'}{f} = \frac{0.5 \times 1.25}{0.25} = 2.5$ or $f' = 2.5\,f$

Thus the focal length increases by a factor of 2.5.

10. (a) Work done in transfering q coulomb of charge from clouds to ground.
$W = Vq = 4 \times 10^6 \times 4 = 16 \times 10^6\,J$
The power of lightning strike is

$P = \frac{W}{t} = \frac{16 \times 10^6}{100 \times 10^{-3}}$
$= 160 \times 10^6\,W = 160\,MW$

11. (a) The intensity of wave varies inversely as the square of distance, $I \propto \frac{1}{r^2}$

Thus, $\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$(\because r_1 = 2m, r_2 = 3m)$

or, $I_1 : I_2 = 9 : 4$

12. (d) The law of conservation of momentum is true in all type of collisions, but kinetic energy is conserved only in elastic collision. The kinetic energy is not conserved in inelastic collision but the total energy is conserved in all type of collisions.

13. (b) When a p-n junction is reverse biased the applied voltage supports the potential barrier as a result the depletion region is increased.
There will be very small reverse current (μ A) due to drifting of minority carriers.

14. (a) As the force is constant therefore slope of surface will change like the surface profile (a).

15. (d) As the batteries wear out, less current flows through filament of flash light so the intensity reduces. Also, the temperature of filament reduces (due to less current) and hence the colour of flash light changed to red because $\lambda \propto \frac{1}{T}$ and red colour has longest wavelength in the spectrum.

16. (a) Electric field lines start from the + ve charge and end on -ve charge.
$|A| > |B|$ the density of field lines is greater at A than B.

17. (b) An alpha particle is ${}_2^4He$

Thus, ${}_{92}^{238}U - {}_2^4He \longrightarrow {}_{90}^{234}Th$

18. (c) After n half-lives $n = \frac{t}{T}$ the number of nuclides left undecayed, $N = N_0\left(\frac{1}{2}\right)^n$

Given, $\frac{N}{N_0} = \frac{1}{16}$

$\therefore \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n$ or $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$

Thus, $n = 4$, i.e, $\frac{t}{T} = 4$ or $t = 4T$

or $t = 4 \times 5730 = 22920$ years

$(\because T = 5730 \text{ years})$

19. (c) When a heavy nucleus emits a beta particle, its mass number remains constant and charge number increases by one.

20. (a) Initially the beat frequency is 4 and the frequency of Q is 250 Hz, thus the frequency of P will be either 254 or 246. When a prong of P is filed, the beat frequency reduces to 2 beats per second and hence frequency of P should be 246.
On filing the frequency of P increases to 248 and it gives 2 beats with Q of frequency 250Hz.

21. (d) According to de-Broglie relation,

$\lambda = \frac{h}{P}$ where P is the momentum and λ is Planck's constant. As the momentum of alpha, beta and gamma rays is the same hence all have the same wavelength.

22. (a) Voltage gain, $A_v = 1000$
In dB, voltage gain
$A_V = 10 \log_{10} 1000 \text{ dB} = (10 \times 3) \log_{10} 10 \text{ dB}$
$= 30 \text{ dB} \qquad (\because \log_{10} 10 = 1)$

23. (c) The entropy is a measure of disorderness of a system. When ice cubes are formed, orderness increases i.e. disorderness decreases and hence entropy decreases.

24. (b) The metal X has a higher coefficient of expansion. When bimetallic strip is placed in a cold bath, there will be greater shrink in the volume of A, as a result the strip will bend towards the left.

25. (b) The given wavelength does not correspond to the green colour of leaf, so it will be absorbed by the leaf and hence it would appear to be black.

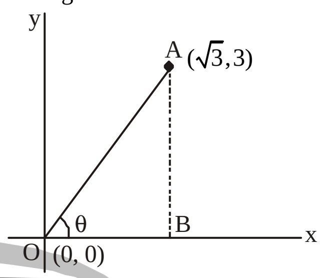
26. (a) The base- emitter of a transistor is made forward biased and the cut-off voltage for silicon is 0.7 V. Thus, the minimum potential difference between the base and emitter to switch the transistor 'on' is 1V.

27. (a) As $T_i = 2T_n - T_o$
Since, the cold junction is kept at 0°C, i.e., $T_o = 0$

Hence, $T_i = 2T_n \qquad \therefore T_n = \frac{T_i}{2}$

28. (b) A nucleus is said to undergo beta decay if it emitts an electron or a positron. In beta minus (e^-) decay, a neutron transforms into a proton within the nucleus and in beta plus (e^+) decay, a proton transforms into neutron inside the nucleus.

29. (c) Let OA represents the path of the particle starting from origin O (0, 0) and reaching point $(\sqrt{3}, 3)$. Let path of the particle makes an angle θ with the x-axis then



$\tan\theta$ = slope of line OA

$= \frac{AB}{OB} = \frac{3}{\sqrt{3}} = \sqrt{3}$ or $\theta = 60°$

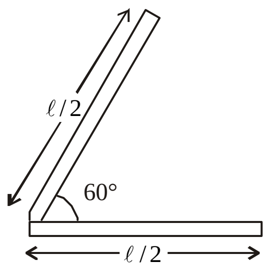
30. (b) The shunt resistance is given by

$S = \left(\frac{I_g}{I - I_g}\right) G$

$= \frac{100}{(750 - 100)} \times 13 = \frac{100 \times 13}{650} = 2\,\Omega.$

31. (d) The two spheres will have the same acceleration (g). The acceleration due to gravity is independent of the mass of the object.

32. (b) The rod is bent in the middle, so that two halves make an angle of 60°. Then each half has a mass of $\frac{M}{2}$ and length $\frac{\ell}{2}$.



Moment of inertia of each part about an axis passing through one end

$$= \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{\ell}{2}\right)^2 = \frac{1}{2}M\ell^2$$

So, the moment of inertia of the whole rod about an axis through its middle point.

$$I = \frac{1}{24}M\ell^2 + \frac{1}{24}M\ell^2 = \frac{1}{12}M\ell^2$$

33. (c) When the boat bounces up, it covers a distance equal to one wave length.

$\lambda = 100$ m, $v = 25$ m/s

$$\therefore \quad \text{time, } t = \frac{\lambda}{v} = \frac{100}{25} = 4\,s$$

34. (c) The pressure in lungs is reduced to 750 mm. The pressure difference between atmospheric pressure and lung pressure

$= (760 - 750)$ mm $= 10$ mm of Hg.

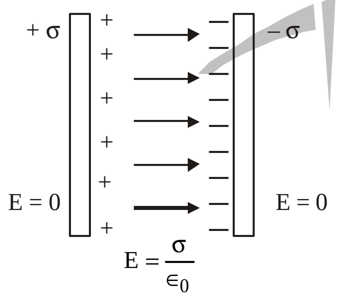
If we can drink water from a glass upto a maximum depth of h then,

$h\rho g = (10 \text{ mm of Hg column})\rho' g$

$h\rho g = (1 \text{ cm})\rho' g$

$h \times 1 = 1 \times 13.6$ or $h = 13.6$ cm.

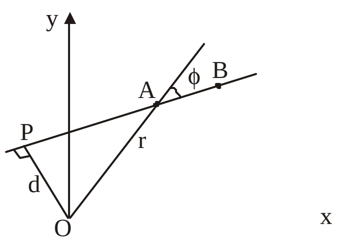
35. (c) The electric field between two parallel thin metal sheets is



$$E = \frac{\sigma}{\epsilon_0} = \frac{26.4 \times 10^{-12}}{8.85 \times 10^{-12}} \approx 3\,N/C$$

36. (b) Angular momentum is

$$\vec{L} = \vec{r} \times \vec{p} = rmv\sin\phi\,(\hat{k})$$



Therefore, the magnitude of L is

$L = mvr \sin\phi = mvd$

where $d = r\sin\phi$ is the perpendicular distance.

As d is same for both the particles, hence $L_A = L_B$

37. (b) Restoring force of the spring $F = |ky|$

$F = ka$ where a is the minimum amplitude of S.H.M.

For the mass to be detached from the pan, $ka = mg$

$$\text{or, } a = \frac{mg}{k} = \frac{2 \times 10}{200}\,m = \frac{1}{10} \times 100\,cm$$

$= 10$ cm.

38. (b) When the electric field becomes zero, the beam of electrons is under influence of magnetic field only. So the electrons move in a circular orbit due to magnetic force acting on them.

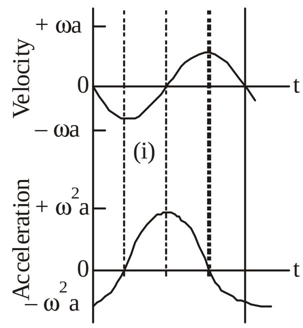
39. (a) The displacement of particle executing SHM is

$x = a\cos(\omega t + \phi)$ ...(i)

$$\text{velocity, } v = \frac{dx}{dt} = -a\omega\sin(\omega t + \phi) \quad \text{...(ii)}$$

Acceleration,

$$A = \frac{dv}{dt} = -a\omega^2\cos(\omega t + \phi) \quad \text{...(iii)}$$



On the basis of graph it is obvious that the acceleration is 90° (0.5π) out of phase with the velocity.

40. (d) Resistance

$$R = \frac{\text{Potential difference}}{\text{Current}} = \frac{V}{i} = \frac{W}{qi}$$

$$\left(\because V = \frac{W}{q}\right)$$

So, dimension of R

$$= \frac{[\text{Dimensions of work}]}{[\text{Dimensions of charge}][\text{Dimension of current}]}$$

$$= \frac{[ML^2T^{-2}]}{[IT][I]} = [ML^2T^{-3}I^{-2}]$$

41. (c) Specific rotation of sugar solution is

$$\alpha = \frac{\text{angle of rotation}}{\text{length of tube} \times \text{concentration}} = \frac{\theta}{\ell \times C}$$

$$\therefore C = \frac{\theta}{\ell \alpha} = \frac{0.4}{0.25 \times 0.01} = 160 \text{ kg/m}^3$$

Thus, purity of sugar solution

$$= \frac{160}{200} \times 100 = 80\%$$

42. (c) The surface area of the disc is maximum and the surface area of cigar is minimum. So 3 < 2 < 1 is the correct option.

43. (a) The focal length of plano-convex lens

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{\infty} + \frac{1}{f}$$

(for plane surface $f_1 = \infty$)

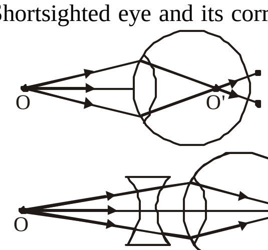
$\therefore F = f$

when a thin concave lens of the same focal length is joined, then the focal length of the combination is

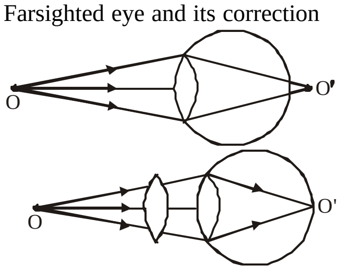
$$\frac{1}{F'} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} - \frac{1}{f} = 0 \quad \text{or, } F' = \infty$$

Thus, the focus shifts to infinity

44. (a) Shortsighted eye and its correction



Farsighted eye and its correction

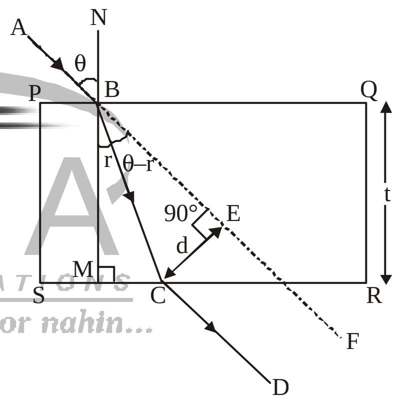


45. (c) (i) The dotted line corresponds to the ideal gas behaviour because there is no change in the value of $\frac{PV}{nT}$ at different temperatures T_1 and T_2.

(ii) The deviation in gas behaviour is more at low temperature and deviation is less at high temperature.
The graphs shows more deviation for temperature T_2 than T_1. Hence, $T_1 > T_2$

(iii) The two curves for different temperatures T_1 and T_2 intersect at dotted line, so the value of $\frac{PV}{nT}$ at that point on the y-axis is same for all gases.

46. (b)



The lateral displacement of emergent ray is given by $d = \frac{t}{\cos r}\sin(\theta - r)$

$$d = \frac{t}{\cos r}(\sin\theta\cos r - \cos\theta\sin r)$$

$$d = t(\sin\theta - \cos\theta\tan r)$$

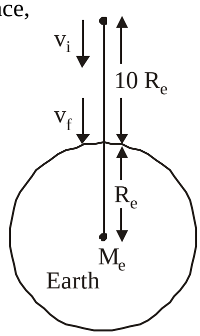
If n is the refractive index of material of slab (glass) w.r.t. air, then $n = \frac{\sin\theta}{\sin r}$

For small angle, $n \approx \frac{\theta}{r}$ or $r = \frac{\theta}{n}$

and d = t (θ –1 . r) [$\sin\theta \approx \theta$ and $\cos\theta \approx 1$ if θ is small]

$$d = t\left(\theta - \frac{\theta}{n}\right) = t\theta\left(1 - \frac{1}{n}\right) \text{ or } d = \frac{t\,\theta(n-1)}{n}$$

47. (c) Applying law of conservation of energy for asteroid at a distance 10 R_e and at earth's surface,



$K_i + U_i = K_f + U_f$ ... (i)

$K_i = \frac{1}{2}mv_i^2$ and $U_i = -\frac{GM_e m}{10R_e}$

$K_f = \frac{1}{2}mv_f^2$ and $U_i = -\frac{GM_e m}{R_e}$

Substituting these values in eq. (i), we get

$$\frac{1}{2}mv_i^2 - \frac{GM_e m}{10R_e} = \frac{1}{2}mv_f^2 - \frac{GM_e m}{R_e}$$

$$\Rightarrow \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + \frac{GM_e m}{R_e} - \frac{GM_e m}{10R_e}$$

$$\Rightarrow v_f^2 = v_i^2 + \frac{2GM_e}{R_e} - \frac{2GM_e}{10R_e}$$

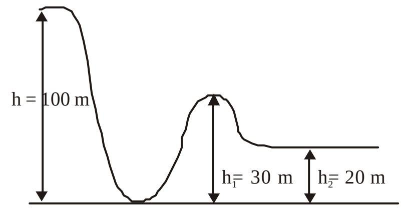
$$\therefore v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 - \frac{1}{10}\right)$$

48. (a) As the proton moves in a direction opposite to the electric field hence work done by the electric field on the proton is negative. The electric potential energy of the proton increases because it moves from low potential to high potential region.

49. (a) Just after closing the switch, the inductor opposes the change in current due to self induction but very soon current attains a steady value.
The effective resistance of circuit 2 is minimum and circuit 1 maximum.
Hence, the current is maximum in circuit 2 and minimum in circuit 1. Thus, $i_2 > i_3 > i_1$.

50. (a) As the charged particle (1) moves along a parabolic path in electric field hence the direction of electric field is upward.
Particle (2) is negative, hence its deflection is downward.
Particle (3) is + ve hence, its deflection is upwards.
Particle (4) is –ve hence its deflection is downwards.

51. (a)



Applying the law of conservation of energy

$(mgh - mgh_2) = \frac{1}{2}mv^2$

or, $mg\,(h - h_2) = \frac{1}{2}mv^2$

or $v^2 = 2g\,(h - h_2) = 2 \times 10\,(100 - 20)$

or, $v = \sqrt{2 \times 10 \times 80} = 40$ m/s

52. (d) Loudness of sound intensity is given by

$\beta = 10\log\left(\frac{I}{I_0}\right)$ ... (i)

After attenuation, $\beta' = 10\log\left(\frac{I'}{I_0}\right)$ ... (ii)

As sound level attenuates by 20 dB
thus $\beta - \beta' = 20$
From equation (i) and (ii),

$$\beta - \beta' = 10\log\left(\frac{I}{I_0}\right) - 10\log\left(\frac{I'}{I_0}\right)$$

$$20 = 10\left[\log\left(\frac{I}{I_0} \times \frac{I_0}{I'}\right)\right] \text{ or } 2 = \log\left(\frac{I}{I'}\right)$$

or antilog (2) $= \frac{I}{I'}$ or $100 = \frac{I}{I'}$ or $I' = \frac{I}{100}$.

53. (a) The parallel plate capacitor consists of n plates hence there are (n – 1) capacitors connected in parallel.
So, the equivalent capacitance will be (n – 1)C

54. (b) An emf of 100 V is generated at 1500 rpm. So, an emf of 120 V will be generated at

$$\frac{1500}{100} \times 120 = 1800 \text{ rpm.}$$

55. (c) Energy received from the sun
$= 2\text{ cal cm}^{-2}(\text{min})^{-1} = 8.4\text{ J cm}^{-2}(\text{min})^{-1}$
Energy of 1 photon corresponding to wavelength 5500 Å

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{5500\times10^{-10}}$$
$= 3.6\times10^{-19}$ J

$\therefore$ Number of photons reaching the earth per cm^2 per minute will be

$$n = \frac{\text{energy received from sun}}{\text{energy of one photon}}$$

$$n = \frac{8.4}{3.6\times10^{-19}} = 2.3\times10^{19}$$

56. (a) Using the relation

$$\omega_0 = \frac{hc}{\lambda_{min}}\left[\begin{array}{l}\because \omega_0 = 35\text{keV}\\ = 35\times10^3\times1.6\times10^{-19}\text{ J}\end{array}\right]$$

or, $$\lambda_{min} = \frac{hc}{\omega_0} = \frac{6.6\times10^{-34}\times3\times10^8}{35\times10^3\times1.6\times10^{-19}}$$
$= 0.355\times10^{-10}$ m
$= 35.5\times10^{-12}$ m $= 35.5$ pm

57. (b) An absorbed dose of 3 Gy corresponds to an absorbed energy per unit mass of 3 J/kg. Let specific heat of human body, is the same as that of water, 4180 J/kg K. Then

$$\Delta T = \frac{Q/m}{c} = \frac{3}{4180} = 7.2\times10^{-4}\text{ K} \approx 700\mu\text{K}.$$

58. (c) The number of potassium atoms remaining at the time of analysis is,
$N_K = N_0 e^{-\lambda t}$ ... (i)
in which t is the age of the rock.
For every potassium atom that decays, an argon atom is produced. Thus, the number of argon atoms present at the line of the analysis is
$N_{Ar} = N_0 - N_K$ ... (ii)
From eq. (i) and (ii), we find that

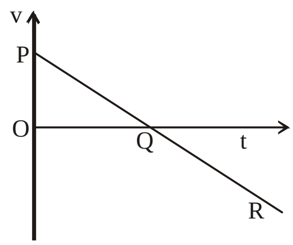
$$\lambda t = \ln\left(1+\frac{N_{Ar}}{N_K}\right),$$

In which N_{Ar}/N_K can be measured. Solving for t

$$t = \frac{T_{1/2}\ln(1+N_{Ar}/N_K)}{\ln 2}$$

$$= \frac{(1.25\times10^9\text{ y})[\ln(1+10.3)]}{\ln 2} = 4.37\times10^9\text{ yr.}$$

59. (a) The velocity of the particle is maximum at the moment it is thrown above. At the highest point its velocity is zero and then it returns back to the initial position. In the graph PQ depicts upward motion and QR part depicts downward motion hence option (a) is correct.



60. (a) $v \propto \sigma^a \rho^b \lambda^c$
Writing dimensions of each term on both sides.
$[M^0LT^{-1}] \propto [MT^{-2}]^a[ML^{-3}]^b[L]^c$
$\propto [M]^{a+b}[L]^{-3b+c}[T]^{-2a}$
Equating the power of M, L and T on both sides,
$a + b = 0$
$-3b + c = 1$
$-2a = -1$

Thus, $a = \frac{1}{2}$, $b = -\frac{1}{2}$, $c = -\frac{1}{2}$

So, $v \propto \sigma^{\frac{1}{2}}\rho^{-\frac{1}{2}}\lambda^{-\frac{1}{2}}$

or, $v^2 \propto \sigma\rho^{-1}\lambda^{-1} \Rightarrow v^2 \propto \frac{\sigma}{\rho\lambda}$

CHEMISTRY

61. (b) SO_2, SCl_4, and SO_3 give oxyacid solution in water.

$$SO_2 + H_2O \longrightarrow \underset{\text{sulphurous acid}}{H_2SO_3}$$

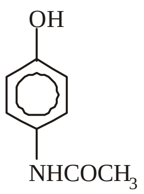
$$SCl_4 + 3H_2O \longrightarrow \underset{\text{sulphurous acid}}{H_2SO_3} + 4HCl$$

$$SO_3 + H_2O \longrightarrow \underset{\text{sulphuric acid}}{H_2SO_4}$$

whereas OF_2 dissolves in water but does not give any oxyacid solution.

62. (a) The tendency of an atom lose electron from its valance shell is known as ionisation potential
Low ionisation potential indicates that element can easily lose electron to form cation.

63. (d) The structure of an important antipyretic, paracetamol is



Thus it is clear that the compound can be obtained by a para substituent.

64. (b) A mixture of amylose and amylopectin is called starch.
Amylose is a water soluble fraction while amylopectin is water insoluble fraction.

65. (a) Alkyl halides give elimination reaction with alcoholic KOH and yield an alkene or alkyne (from dihalides) eg,

$$\underset{1,2-\text{dibromoethane}}{Br-CH_2-CH_2-Br} \xrightarrow[\Delta]{\text{alc.KOH}} \underset{\text{acetylene}}{\overset{sp}{CH} \equiv \overset{sp}{CH}}$$

Hence, product has both *sp*-hybridised carbon.

66. (d) Permutit is hydrated Sodium aluminium silicate $Na_2Al_2Si_2O_8.xH_2O$. It exchanges its sodium ions for divalent ions such as Ca^{2+} and Mg^{2+}.

$$Na_2Al_2Si_2O_8 + CaCl_2 \rightarrow CaAl_2Si_2O_8 + 2NaCl$$

$$Na_2Al_2Si_2O_8 + MgSO_4 \rightarrow MgAl_2Si_2O_8 + Na_2SO_4$$

Permutit when fully exhausted can be regenerated by treating with 10% solution of sodium chloride

$$Ca-\text{permutit} + 2NaCl \rightarrow 2Na-\text{permutit} + CaCl_2$$

$$Mg-\text{permutit} + 2NaCl \rightarrow 2Na-\text{permutit} + MgCl_2$$

It is most efficient method to get water with zero degree hardness.

67. (c) Lyophilic colloids are those colloids which from colloidal solution in contact with water. Starch is an example of lyophilic (water loving) colloidal solution.

68. (c) Xenon hexafluoride reacts with silica to form $XeOF_4$ as

$$2XeF_6 + SiO_2 \longrightarrow \underset{'X'}{XeOF_4} + SiF_4$$

The oxidation state of xenon in $XeOF_4$ is calculated as $\overset{x}{Xe}\overset{-2}{O}\overset{-1}{F_4}$

$x + (-2) + 4 \times (-1) = 0$
$x - 2 - 4 = 0$
Thus O.S of Xe in compound X is + 6.

69. (a) When oxyhaemoglobin changes to deoxyhaemoglobin, Fe^{2+} ion changes from diamagnetic to paramagnetic.

70. (d) In atomic reactors helium gas is used.
It is noninflammable. It has low density and its lifting power is 92% that of hydrogen and used to lift weather balloons and airships.

71. (d) $KMnO_4$ reacts with oxalic acid according to the following equation

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

Eq. mass of $KMnO_4$

$$= \frac{\text{mol.mass}}{\text{change in O.N.}} = \frac{\text{mol.mass}}{7-2}$$

$$N_{KMnO_4} = 5 \times \text{molarity} = 5 \times 10^{-4}$$

Eq. mass of $C_2O_4^{2-}$

$$= \frac{\text{mol.mass}}{2(4-3)} = \frac{\text{mol.mass}}{2}$$

$$N_{C_2O_4^{2-}} = 2 \times \text{molarity} = 2 \times 10^{-2}$$

$N_1V_1 = N_2V_2$
$5 \times 10^{-4} \times V_1 = 2 \times 10^{-2} \times 0.5$

$$V_1 = \frac{2 \times 10^{-2} \times 0.5}{5 \times 10^{-4}} = 20\,L$$

72. (b) The atomic number of cesium is 55. The electronic configuration of cesium atom is
${}_{55}Cs = 1s^2, 2s^2 2p^6, 3s^2 3p^6, 4s^2, 3d^{10} 4p^6, 5s^2, 4d^{10}, 5p^6, 6s^1$
The electronic configuration of cesium ion will be
$Cs^+ = 1s^2, 2s^2 2p^6, 3s^2 3p^6, 4s^2, 3d^{10} 4p^6, 5s^2, 4d^{10}, 5p^6, 6s^0$

Therefore,
the total number of *s* electrons = 10
the total number of *p* electrons = 24,
the total number of *d* electrons = 20.

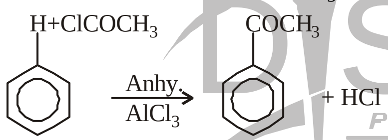
73. (b) Nitrobenzene is reduced by Zn and alcoholic KOH into hydrazobenzene.

$$2C_6H_5NO_2 + 10H \xrightarrow{\text{Zn+alc.KOH}} \underset{\text{hydrazobenzene}}{C_6H_5-NH-NH-C_6H_5} + H_2O$$

74. (a) Chloroform on reaction with sodium ethoxide gives ethyl ortho formate $[CH(OC_2H_5)_3]$.

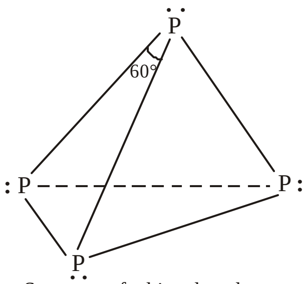
$$CHCl_3 + 3C_2H_5ONa \longrightarrow \underset{\text{ethylorthoformate}}{CH(OC_2H_5)_3} + 3NaCl$$

75. (c) Acetophenone (is prepared by substitution), whereas other compounds are prepared by addition reactions. Acetophenone is prepared by Friedel Craft's reaction when benzene reacts with acetyl chloride in presence of anhy. $AlCl_3$ as



acetophenone

76. (b) Molecular weight determination suggests that white phosphorus has the molecular formula P_4 both in solid and vapours states. The four atoms of the molecule are at the corners of a regular tetrahedron. The P – P bond distance on P_4 molecules is 221 pm and the P – P – P bond angles are of 60°. However, theoretically the pure P – P overlap would give bond angle of 90°; hence the low angles in P_4 molecule suggests that the structure is under strain and hence unstable.



Structure of white phosphorus

77. (b) The electron affinity (in kJ/mol) are given as Fluorine = 332.6; Chlorine = 348.5 Bromine = 324.7; Iodine = 295.5
The electron affinity value is highest for the chlorine. Hence, the correct order of electron affinity will be
$Cl_2 > F_2 > Br_2$

78. (c) Radioactive disintegration is a Ist order reaction, hence,

$$k = \frac{2.303}{t}\log\frac{a}{a-x}$$

$$\therefore \quad k = \frac{2.303}{15}\log\frac{100}{100-50}$$

$$\text{also, } k = \frac{2.303}{t}\log\frac{100}{100-99}$$

$$\therefore \frac{2.303}{15}\log\frac{100}{50} = \frac{2.303}{t}\log\frac{100}{1}$$

$$\frac{1}{15}\log 2 = \frac{1}{t}\log 100 \Rightarrow \frac{0.3010}{15} = \frac{2}{t}$$

$$t = \frac{2\times 15}{0.3010} = 99 \text{ min}$$

79. (b) Change in temperature = 80 – 30 = 50°C
As we know that for every 10°C increase in temperature, rate of reaction doubles.
Thus, for 50°C increase in temperature, increase in rate of reaction = 2^5 = 32 times.

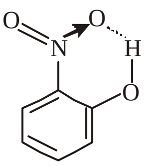
80. (d) BCl_3 molecule has sp^2 hybridization, in which three sp^2 half filled hybrid orbitals are planar and oriented at an angle of 120° to each other. NH_3 and H_2O has sp^3 hybridisation, thus should give a bond angle 109° 28'. However, the observed bond angle is 107° in NH_3 and 104.5° in H_2O. This is due to the presence of lone pair of electrons. AsH_3 has smaller bond angle than NH_3 due to less electronegativity of As compared to N.

81. (c) Physical adsorption decreases with increase in temperature, whereas chemisorption first increases and then decreases with increase in temperature.

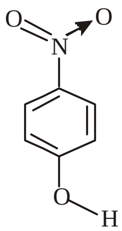
82. (c) Acetylene when react with formaldehyde in presence of copper acetylide catalyst form an addition product.

$$\underset{\text{acetylene}}{CH \equiv CH} + 2HCHO \longrightarrow \underset{\text{2-butyne-1,4-diol}}{OHH_2C-C \equiv C-CH_2OH}$$

83. (c) *Ortho*-and *para*-nitrophenol is separated by distillation because *p*-nitrophenol has higher boiling point than *o*-nitrophenol due to H-bonding.



o-Nitrophenol
More volatile



p-Nitrophenol
Less volatile

84. (a) In S_{N^1} mechanism, carbocations are formed as intermediate, hence more the stability of the intermediate carbocation, greater are chances for their formation and hence more reactive will be the parent alkyl halide for S_{N^1} reaction. Hence the order of reactivity of alkyl halides toward S_{N^1} reaction follows the order : 3° > 2° > 1°

$$\underset{\text{t-butylchloride}}{H_3C-\overset{\overset{CH_3}{|}}{\underset{\underset{CH_3}{|}}{C}}-Cl} \xrightarrow[-Cl]{\text{slow}} \underset{\text{carbocation}}{(H_3C)_3C^+}$$

$$\xrightarrow[\text{fast}]{+OH^-} \underset{\text{t-butyl alcohol}}{(H_3C)_3COH}$$

85. (c) For isotonic solutions

$$C_1 = C_2 \text{ or } \frac{n_1}{V_1} = \frac{n_2}{V_2}$$

$$\frac{W_1}{m_1 V_1} = \frac{W_2}{m_2 V_2}$$

$V_1 = 1L, V_2 = 100\text{ mL} = 0.1\text{ L}$

$$\frac{W_1}{m_1 V_1} = \frac{W_2}{m_2 V_2} \Rightarrow \frac{W_1}{60 \times 1} = \frac{10}{342 \times 0.1}$$

$W_1 = 17.54$ g/L

86. (c) According to Fajan's rule,
Polarising power of cation

$$= \frac{\text{chargeoncation}}{\text{radiusofcation}}$$

Therefore, as the size of cation decreases, its polarising power increases. Hence, Cu^{2+} polarise Cl^- ions more than Cu^+. Therefore, $CuCl_2$ has more covalent character and hence, its boiling point is less.

87. (c) For a body centred cubic lattice

$$\text{radius (r)} = \frac{\sqrt{3}}{4}a = 0.433a$$

Therefore, radius of $Na^+ = 0.433 \times 4.29$
$= 1.8575$

88. (b) Most probable velocity $= \sqrt{\frac{8RT}{\pi m}}$

T = (27 + 273) = 300 K
Most probable velocity (H_2)

$$= \sqrt{\frac{8 \times 8.314 \times 10^7 \times 300}{3.14 \times 2}} = 17.8 \times 10^4 \text{ cm/s.}$$

89. (d) The paramagnetic behaviour depends upon the no. of unpaired electron. More the no. of unpaired electron more is the paramagnetic character. In the given species no. of unpaired as follows
$Mn^{++} = [Ar]\,3d^5$ = 5 unpaired electron
$Fe^{++} = [Ar]\,3d^6$ = 4 unpaired electron
$Cu^{++} = [Ar]\,3d^9$ = 4 unpaired electron

90. (c) Au is a member of 5d-series. Fe, Co and Cu all are the members of 3d-series.

91. (b) As we know that, $t_{1/2} \propto \frac{1}{a^{n-1}}$

$$\text{Thus } \frac{(t_{1/2})_1}{(t_{1/2})_2} = \left[\frac{a_2}{a_1}\right]^{n-1}$$

where, n = order of the reaction
On substituting the values

$$\frac{0.1}{0.8} = \left[\frac{50}{400}\right]^{n-1}$$

On taking log both sides

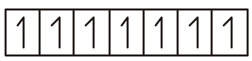
$$\log\frac{0.1}{0.8} = (n-1)\log\frac{50}{400}$$

$$\log\frac{1}{8} = (n-1)\log\frac{1}{8}$$

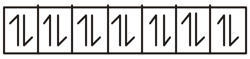
n – 1 = 1 or n = 2

92. (a) Gadolinium (Z = 64) : $[Xe]\,4f^7, 5d^1, 6s^2$

$4f^7$

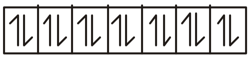


Lutetium (Z = 71) : [Xe] $4f^{14}, 5d^1, 6s^2$



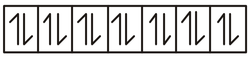
$4f^{14}$

Lawrencium (Z = 103): [Rn] $5f^{14}, 6d^1, 7s^2$



$5f^{14}$

Tantalum (Z = 73) : [Xe] $4f^{14}, 5d^3, 6s^2$



$4f^{14}$

Hence, Gadolinium has got incompletely filled *f* subshell.

93. (d) Galvanic cell is

$Cu\,(s)\,|\,Cu^{2+}\,(aq)\,||\,Hg^{2+}\,(aq)\,|\,Hg\,(\ell)$

In the above cell, oxidation of copper and reduction of mercury takes place. Its cell reaction is written as

$$\underbrace{Cu(s) + \overbrace{Hg^{2+}(aq) \longrightarrow Cu^{2+}}^{\text{reduction}}}_{\text{oxidation}}(aq) + Hg(\ell)$$

94. (b) $E_{cell} = E^\circ_{cell} + \frac{0.059}{n}\log[M^{n+}]$

On substituting the values given

$$E_{cell} = 0.34 + \frac{0.059}{2}\log 10^{-2}$$

$$= 0.34 + \frac{0.059}{2} \times -2 = +0.281\text{ V}$$

95. (d) Graphite, unlike diamond, has a two dimensional sheet llike structure. The adjacent layers of sheet are held together by weak vander Waal forces. Here, each carbon atom is in sp^2 hybridised state and is thus attached to three other carbon atoms by three σ bonds forming a hexagonal planar structure. The fourth electron present in an unhybridised *p* orbital of each carbon atom of a hexagonal unit then overlaps with each other to form a π bond. Hence the C – C **bond length in graphite is shorter (142 pm) than that of dimond (154 pm)**. Now since *p*-electrons (π electron) are free to move throughout the entire layers, **graphite is a good conductor of electricity**.

96. (a)

$$X(g) \rightleftharpoons Y(g) + z(g)$$

	X	Y	Z
t = 0	1	0	0
t_{eq}	0.5	0.5	0.5

$$P_X = \frac{0.5}{1.5}p \quad P_Y = \frac{0.5}{1.5}p \quad P_Z = \frac{0.5}{1.5}p$$

$$\because \quad K_p = \frac{p_Y \cdot p_Z}{p_X}$$

$$1 = \frac{\frac{p}{3} \times \frac{p}{3}}{\frac{p}{3}} \quad \therefore \quad p = 3\text{ atm.}$$

Partial pressure of X $= \frac{p}{3} = \frac{3}{3} = 1$ atm.

97. (a) Ozone layer is found in the stratosphere region of atmosphere. It shield the earth from the harmful U. V. radiations from the sun. The U. V. radiations cause skin cancer, cataract of eye, and harmful to vegetation.

98. (c) H_2O_2 can be prepared by electrolysis of 50% H_2SO_4. In this method, hydrogen is liberated at cathode.

$$2H_2SO_4 \rightleftharpoons H_2 + \underset{\substack{\text{Persulphuric acid}\\ \text{(Marshall's acid)}}}{H_2S_2O_8}$$

$$H_2S_2O_8 + H_2O \rightarrow H_2SO_4 + \underset{\substack{\text{Permonosulphuric acid}\\ \text{(Caro's Acid)}}}{H_2SO_5}$$

$$H_2SO_5 + H_2O \rightarrow H_2SO_4 + H_2O_2$$

Mechanism : $H_2SO_4 \rightleftharpoons H^+ + HSO_4^-$

At anode : $2\,HSO_4^- \rightarrow H_2S_2O_8 + 2e^-$

At cathode : $2\,H^+ + 2e^- \rightarrow H_2$

99. (b) Alkali metals are electropositive, hence they can reduce CO_2.

$$4Na + CO_2 \longrightarrow 2Na_2O + C$$

100. (d) $N_2\,(g) + 3H_2(g) \rightleftharpoons 2NH_3\,(g) + 22\text{ kcal}$

The given reaction is an example of exothermic reaction and for exothermic reaction

$E_{a(F.R)} + \Delta H = E_{a\,(B.R)}$

$E_{a\,(F.R)} = 50$ kcal

$E_{a\,(B.R)} = 50 + 22 = 72$ kcal.

101. (c) Aldehydes are reduced by $LiAlH_4$ to alcohols and alcohols are oxidised by copper to give aldehydes.

$$\underset{(X)}{CH_3CHO} \xrightarrow[{[2H]}]{LiAlH_4} \underset{(Y)}{CH_3CH_2OH}$$

$$\xrightarrow[\Delta]{Cu} \underset{(X)}{CH_3CHO}$$

102. (c) In Wurtz reaction alkyl halide reacts with sodium in presence of dry ether to give alkanes eg,

$$C_2H_5\boxed{Cl + 2Na + Cl}C_2H_5 \xrightarrow[\text{(dryether)}]{}$$

$$\underset{\text{butane}}{C_4H_{10}} + 2NaCl$$

In Wurtz reaction wet ether is not used because wet ether destroy the sodium metal.

103. (b) Aldehydes and Ketones having at least one α-hydrogen atom in presence of dil. alkali give β-hydroxy aldehyde or β-hydroxy ketone, which on heating gives α,β-unsaturated carbonyl compound.

$$\underset{\text{Acetaldehyde}}{CH_3-\overset{\overset{O}{||}}{\underset{\underset{H}{|}}{C}}} + HCH_2CHO$$

$$\xrightarrow{\text{dil.NaOH}} \underset{\text{Aldol}}{CH_3-\overset{\overset{OH}{|}}{\underset{\underset{H}{|}}{C}}-CH_2-CHO}$$

$$\xrightarrow[-H_2O]{\Delta} \underset{\text{Crotonaldehyde}}{CH_3-CH=CH.CHO}$$

104. (c) We know that

$$\Delta E = hc.R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

For lowest energy, of the spectral line in Lyman series, $n_1 = 1$, $n_2 = 2$

Hence,

$$\Delta E = hc.R\left[\frac{1}{1^2}-\frac{1}{2^2}\right] \text{ or } \Delta E = \frac{3hcR}{4}$$

105. (c) $N = \dfrac{N_1V_1 + N_2V_2}{V_1 + V_2}$ $(\because \text{For HCl } N = M)$

$$= \frac{0.015\times100 + 0.005\times100}{100+100}$$

$$= \frac{1.5+0.5}{200} = \frac{1}{100} = 10^{-2}$$

Resulting solution is acidic in nature.

Then, $[H^+] = 10^{-2}$

$pH = -\log[H^+]$

$$= \log\frac{1}{[H^+]} = \log\frac{1}{10^{-2}} = 2\log 10 = 2$$

106. (b) The reactions in which energy is released are known as exothermic reactions eg,

$$N_2 + 3H_2 \longrightarrow 2NH_3 + 92kJ$$

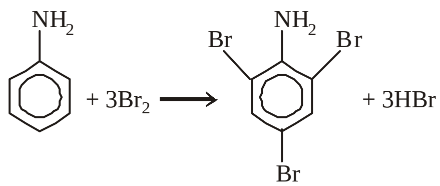
In this equation energy is released, so it is an example of exothermic reaction.

107. (d) The formula of carnalite is $KCl.\ MgCl_2.6H_2O$. In this formula only potassium gives colour (lilac) to flame, whereas magnesium does not give flame colouration.

108. (d) Due to intermolecular hydrogen bonding in ethanol, it exist as associated molecule whereas no hydrogen bonding occurs in diethyl ether. Therefore, ethanol has higher boiling point than diethyl ether.

Therefore, the b.p of ether will be 34°C

109. (b) Aniline reacts with Br_2 to give 2, 4, 6-tribromoaniline not bromoaniline as :



2,4,6-tribromo aniline

110. (a) The values of four quantum numbers given represents following characteristics.

$n = 4 \rightarrow$ valence electron is present in fourth shell.

$\ell = 0 \rightarrow$ valence electron is present in s subshell.

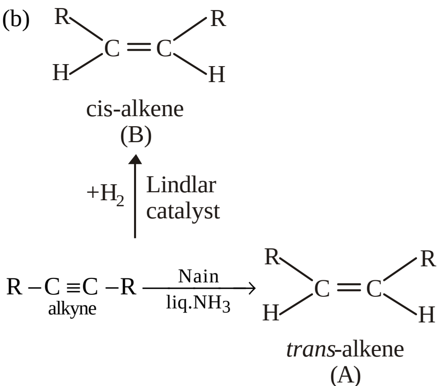
$m = 0 \rightarrow$ valence electron is present in orbital of s. subshell.

$s = +\dfrac{1}{2} \rightarrow$ spinning of electron is clockwise.

Therefore from the above points it is clear that the valence electron is present in 4s as

$4s^1$, which indicates that this is an alkali metal present in 4th period and IA group i.e. K.

111. (b)



112. (b) **Lanthanide contraction -** There is a steady decrease in the radii as the atomic number of the lanthanide elements increases. For every additional proton added in nucleus the corresponding electron goes to 4*f* subshell. The shape of *f-orbitals* is very much diffused and they have poor sheilding effect. The effective nuclear charge increases which causes the contraction in the size of electron charge cloud. This contraction in size is **quite *regular*** and known as ***Lanthanide contraction***.

113. (d) Nylon-6 is used in the manufacture of tyre cord.

114. (c) Synthetic petrol is prepared by Fischer-Tropsch process.

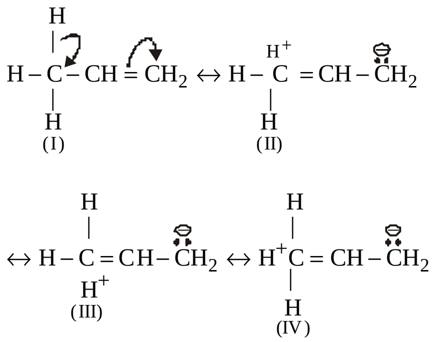
$$\underset{\text{Red hot coke}}{C} + \underset{\text{Steam}}{H_2O} \xrightarrow{1470K} \underbrace{CO + H_2}_{\text{watergas}}$$

$$\underbrace{CO + H_2}_{\text{watergas}} + H_2 \xrightarrow[\text{oxides of Cu, Cr and Zn}]{285-290\,C} \text{Mixture of liquid hydrocarbons}$$

115. (b) Ethylene glycol reacts with excess of PCl_5 to give ethylene chloride.

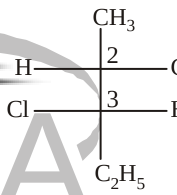
$$\underset{\text{ethylene glycol}}{\begin{array}{c} CH_2OH \\ | \\ CH_2OH \end{array}} + 2PCl_5 \longrightarrow \underset{\text{1, 2-dichloro ethane or ethylene chloride}}{\begin{array}{c} CH_2Cl \\ | \\ CH_2Cl \end{array}} + 2POCl_3 + 2HCl$$

116. (a) Orbital interaction between the σ-bonds of a substituent group and a neighbouring π-orbital is known as hyperconjugation.



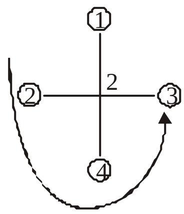
Atoms with higher atomic numbers receive higher priorities.

117. (b) In



Therefore, priority of the groups for C-2 carbon is

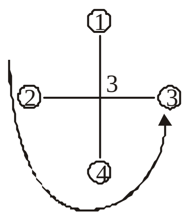
Cl > C-3 carbon > CH_3 > H
① ② ③ ④



S-configuration *i.e.* 2 *S*

Priority of groups for C-3 carbon is

Cl > C-2 carbon > C_2H_5 > H
① ② ③ ④



S-configuration *i.e.* 3 *S*

118. (d) Given

$$C_2H_2(g) + \frac{5}{2}O_2(g) \longrightarrow 2CO_2(g) + H_2O(\ell); \quad \text{... (i)}$$

$\Delta G° = -1234$ kJ

$C(s) + O_2(g) \rightarrow CO_2(g); \Delta G° = -394$ kJ ... (ii)

$$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(\ell); \Delta G° = -237 \text{ kJ} \quad \text{... (iii)}$$

Operate 2 × (ii) + (iii) – (i)

$2C(s) + H_2(g) \longrightarrow C_2H_2(g)$

$\Delta G° = 2(-394) + (-237) - (-1234)$

$\Delta G° = 209$ kJ

119. (b) Let the solubility of AgCl is x

$$\underset{}{AgCl} \rightleftharpoons \underset{x}{Ag^+} + \underset{x}{Cl^-}$$

$[Cl^-]$ from NaCl = 0.2

Concentration of $Cl^- = x + 0.2$

$K_{sp} = x(x + 0.2)$

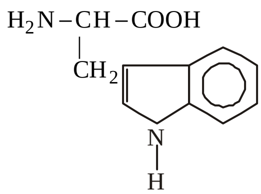
$1.8 \times 10^{-10} = x^2 + 0.2x$

(x is very small as AgCl is sparingly soluble in water, thus $x^2 <<< 1$)

$1.8 \times 10^{-10} = 0.2x$

$$x = \frac{1.8 \times 10^{-10}}{0.2} = 9.0 \times 10^{-10} M$$

120. (d) Tryptophan is a heterocyclic amino acid.



MATHEMATICS

121. (b) Given equation is $1 - \cos\theta = \sin\theta \sin\frac{\theta}{2}$

$$\Rightarrow 2\sin^2\frac{\theta}{2}\left[1 - \cos\frac{\theta}{2}\right] = 0$$

$$\Rightarrow \sin\frac{\theta}{2} = 0 \text{ or } \cos\frac{\theta}{2} = 1 = \cos 0$$

$$\Rightarrow \frac{\theta}{2} = k\pi + 0 \text{ or } \frac{\theta}{2} = 2k\pi + 0, k \in I$$

$$\Rightarrow \theta = 2k\pi \text{ or } \theta = 4k\pi, k \in I$$

122. (c) Given expression is

$$\frac{e^{7x} + e^{3x}}{e^{5x}} = e^{2x} + e^{-2x}$$

We know $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

$$\therefore e^{2x} = 1 + \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + ... \infty \quad \text{... (i)}$$

and

$$e^{-2x} = 1 - \frac{2x}{1!} + \frac{(2x)^2}{2!} - \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} - ... \infty \quad \text{.... (ii)}$$

By adding equations (i) and (ii), we get

$$e^{2x} + e^{-2x} = 2\left[1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + ... \infty\right]$$

Therefore, the constant term is 2.

123. (c) Consider the expression

$$\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \frac{C_6}{7} + ...$$

Putting the values of $C_0, C_2, C_4 ...$, we get

$$= 1 + \frac{n(n-1)}{3.2!} + \frac{n(n-1)(n-2)(n-3)}{5.4!} + ...$$

Multiply and divide by n + 1

$$= \frac{1}{n+1}\left[(n+1) + \frac{(n+1)n(n-1)}{3!} + \frac{(n+1)n(n-1)(n-2)(n-3)}{5!} + ...\right]$$

Put n + 1 = N in above expression

$$= \frac{1}{N}\left[N + \frac{N(N-1)(N-2)}{3!} + \frac{N(N-1)(N-2)(N-3)(N-4)}{5!} + ...\right]$$

$$= \frac{1}{N}[{}^NC_1 + {}^NC_3 + {}^NC_5 + ...]$$

$$= \frac{1}{N}[2^{N-1}] = \frac{2^n}{n+1} \quad [\because N = n+1]$$

124. (c) The number lying between 999 and 10000 are four digit numbers.

The four digit numbers formed by the help of the digits 0, 2, 3, 6, 7, 8 are ${}^6P_4 = \frac{6!}{2!} = 360$

But these 360 numbers involves those nos. which begin from 0. and those numbers are three digit numbers.
Taking initial digit 0, the number of ways to fill remaining 3 places from five digits 2, 3, 6, 7, 8 are $^5P_3 = 60$
Therefore, the required numbers = 360 – 60
= 300

125. (b) Let the two numbers be a and b

Given A.M. = $\frac{a+b}{2} = 9$ and

G.M. = a . b = 16 $\Rightarrow$ a + b = 18 and ab = 16
Therefore, the required quadratic equation is $x^2 - 18x + 16 = 0$

126. (b) Since, the line drawn from the points A (4, – 1, 2) and B (–3, 2, 3). So the dr's of the line. AB is (4 + 3, – 1 –2, 2 – 3)
i.e., (7, – 3, –1)
Since the line meet a plane at right angle. Therefore dr's of line AB is proportional to the normal of the plane
Also, line AB meets a plane at a point (–10, 5, 4).
$\therefore$ Required equation of plane is
$7x - 3y - z + 89 = 0$

127. (d) Given quadratic equation is $2x^2 + 7x + c = 0$.
Let α and β be the roots of the equation $2x^2 + 7x + c = 0$,

$\therefore$ Sum of roots = $\alpha + \beta = -\frac{7}{2}$ and

Product of roots = $\alpha\beta = \frac{c}{2}$

Given, $|\alpha^2 - \beta^2| = \frac{7}{4}$

$\Rightarrow \alpha^2 - \beta^2 = \pm\frac{7}{4}$ (By defn. of modulus)

$\Rightarrow (\alpha - \beta)(\alpha + \beta) = \pm\frac{7}{4}$

We know $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow (\alpha - \beta)^2 = \left(\frac{-7}{2}\right)^2 - \frac{4c}{2} = \frac{49}{4} - 2c$

$\Rightarrow (\alpha - \beta) = \sqrt{\frac{49}{4} - 2c}$

$\Rightarrow -\frac{7}{2}\sqrt{\frac{49}{4} - 2c} = \pm\frac{7}{4} \Rightarrow c = 6$

128. (c) Consider, $\cos^2\left(\frac{\pi}{3} - x\right) - \cos^2\left(\frac{\pi}{3} + x\right)$
By using $(a^2 - b^2) = (a - b)(a + b)$, we have

$= \left\{\cos\left(\frac{\pi}{3} - x\right) + \cos\left(\frac{\pi}{3} + x\right)\right\}$

$\left\{\cos\left(\frac{\pi}{3} - x\right) - \cos\left(\frac{\pi}{3} + x\right)\right\}$

By using

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

and

$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$,

We have

$= \left\{2\cos\frac{\pi}{3}\cos x\right\}\left\{2\cos\frac{\pi}{3}\sin x\right\}$

$= \left\{2\sin\frac{\pi}{3}\cos\frac{\pi}{3}\right\}$ {2 sin x cos x}

$= \sin\frac{2\pi}{3}\sin 2x$

(By using $\sin 2\theta = 2\sin\theta\cos\theta$)

$= \sin\left(\pi - \frac{\pi}{3}\right)\sin 2x$

$= \sin\frac{\pi}{3}\sin 2x = \frac{\sqrt{3}}{2}\sin 2x$.

Since, maximum value of sin 2x is 1
Hence, max value of

$\cos^2\left(\frac{\pi}{3} - x\right) - \cos^2\left(\frac{\pi}{3} + x\right)$ is $\frac{\sqrt{3}}{2}$.

129. (d) Let the first circle be
$S_1 : x^2 + y^2 - 6x - 2y + 1 = 0$ and second be $S_2 : x^2 + y^2 + 2x - 8y + 13 = 0$
On comparing given equations with
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
We get centres of the circles S_1 and S_2 are c_1 (3, 1) and c_2 (–1, 4) and corresponding radii are $r_1 = 3$ and $r_2 = 2$.
Now, $c_1c_2 = \sqrt{25} = 5$
Also, $r_1 + r_2 = 5$
$\therefore c_1c_2 = r_1 + r_2$
Hence, two circles touch each other externally.

130. (a) Let $S = \frac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + \ldots \infty$

$\therefore$ nth term $T_n = \frac{1}{(2n-1)(2n)(2n+1)}$

$= \frac{1}{2}\left[\frac{1}{2n-1} - \frac{1}{2n}\right] - \frac{1}{2}\left[\frac{1}{2n} - \frac{1}{2n+1}\right]$

Now, By putting $n = 1, 2, 3, \ldots$ we get $T_1, T_2, T_3 \ldots$

$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{2}\right] - \frac{1}{2}\left[\frac{1}{2} - \frac{1}{3}\right]$

$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{4}\right] - \frac{1}{2}\left[\frac{1}{4} - \frac{1}{5}\right]$

$\therefore$ Sum $= S_n$
$= T_1 + T_2 + T_3 + \ldots + T_n + \ldots$

$= \frac{1}{2}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots\right]$

$- \frac{1}{2}\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \ldots\right]$

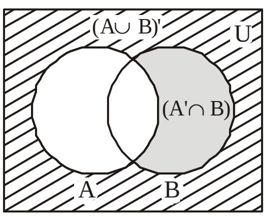
$= \frac{1}{2}\log_e(1+1) + \frac{1}{2}\left[-1 + \left\{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right\}\right]$

By using

$\log(1+x) = x \quad \frac{x^2}{2} + \frac{x^3}{3} \quad \frac{x^4}{4} + .$

$= \log_e 2 - \frac{1}{2}$

131. (a) From Venn Diagram we have



$\therefore (A \cup B)' \cup (A' \cap B) = A'$

Alternative method :

Consider

$(A \cup B)' \cup (A' \cap B) = (A' \cap B') \cup (A' \cap B)$

Let $x \in (A' \cap B') \cup (A' \cap B)$

$\Rightarrow x \in A' \cap B'$ or $x \in A' \cap B$

$\Rightarrow x \in A'$ and $x \in B'$ or $x \in A'$ and $x \in B$

$\Rightarrow x \in A'$ ($\because$ It is not possible that x belongs to both B and B')

132. (a) Suppose $y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$

$\Rightarrow \sin y = \log_3\left(\frac{x}{3}\right)$

As we know $-1 \le \sin y \le 1$

$\Rightarrow -1 \le \log_3\left(\frac{x}{3}\right) \le 1 \Rightarrow \frac{1}{3} \le \frac{x}{3} \le 3$

$\Rightarrow 1 \le x \le 9 \Rightarrow x \in [1, 9]$

133. (a) Let $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$

$\therefore$ Discriminant $= b^2 - 4ac < 0$

$\Rightarrow 4a^2 - 4(10 - 3a) < 0$

$\Rightarrow 4a^2 - 40 + 12a < 0 \Rightarrow a^2 + 3a - 10 < 0$

$\Rightarrow (a+5)(a-2) < 0 \Rightarrow -5 < a < 2$

134. (d) Let $I = \int_0^2 \frac{x^3 dx}{(x^2+1)^{3/2}}$

Put $t = x^2 + 1$

such that $dt = 2x dx$

when $x = 0$, $t = 1$ and $x = 2 \Rightarrow t = 5$

$\therefore \quad I = \int_0^2 \frac{x_3}{(x^2+1)^{3/2}} dx = \frac{1}{2}\int_1^5 \frac{(t-1)}{t^{3/2}} dt$

$= \frac{1}{2}\int_1^5 [t^{-1/2} - t^{-3/2}] dt$

$= \frac{1}{2}\left[2\sqrt{t} + 2\frac{1}{\sqrt{t}}\right]_1^5 = \frac{6 - 2\sqrt{5}}{\sqrt{5}}$

135. (d) Let $I = \int [\sin(\log x) + \cos(\log x)] dx$

$= \int \sin(\log x) dx + \int \cos(\log x) dx \quad \ldots (1)$

Consider $\int \sin(\log x)dx$

$= \int 1 \times \sin(\log x)dx$

$= \sin(\log x) . x - \int \frac{\cos(\log x)}{x} . x dx$

$= x \sin(\log x) - \int \cos(\log x)dx$

Now, put this value in Eq^n (1), we get

$= x \sin(\log x) - \int \cos(\log x)dx$

$+ \int \cos(\log x)dx + c$

$= x \sin(\log x) + c$

136. (c) Since, given function $f(x) = x(x+3)\ e^{-\frac{x}{2}}$ satisfies all the conditions of Rolle's theorem in [–3, 0]

$\therefore$ there exist $c \in (-3, 0)$ such that $f'(c) = 0$

Now $f(x) = x(x+3)e^{-(1/2)x}$

$\therefore \quad f'(x) = x^2 + 3x)e^{-(1/2)x} . \left(-\frac{1}{2}\right)$

$+ (2x+3)e^{-(1/2)x}$

$= -\frac{1}{2} e^{-(1/2)x}\{x^2 - x - 6\}$

Now, $f'(c) = 0$

$\Rightarrow -\frac{1}{2}e^{-(c/2)}\{c^2 - c - 6\} = 0 \Rightarrow c = 3, -2$

But $c = 3 \notin [-3, 0] \qquad \therefore c = -2$

137. (a) Given function is $f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$

Replace x by – x

$\therefore \quad f(-x) = \frac{e^{-2x} - 1}{e^{-2x} + 1}$

$= \frac{e^{-2x}(1 - e^{2x})}{e^{-2x}(1 + e^{2x})} = \frac{1 - e^{2x}}{1 + e^{2x}}$

$\Rightarrow f(-x) = -f(x)$

Note : A function f (x) is said to be an odd function if $f(-x) = -f(x)$.

$\therefore$ From the note, we have

f (x) is an odd function.

$\therefore$ Option 'c' is not correct.

Now, we check f (x) is increasing or decreasing.

$f'(x) = \frac{4e^{2x}}{(1+e^{2x})^2} > 0, \forall x \in R$

Since, $f'(x) > 0, \forall x \in R$

$\therefore \quad$ f (x) is an increasing function.

138. (c) Let h feet be the complete height of the house and t be the total time taken by the stone then we know $h = \frac{1}{2}gt^2$ ....(1)

Since, stone dropped from the top of the house passes the lowest storey in $\frac{1}{4}$ s.

$\therefore \quad h = \frac{1}{4}g\left(t - \frac{1}{4}\right)^2$

Now, the height of the lowest storey is 20 ft. Therefore, we have

$20 = \frac{1}{2}gt^2 - \frac{1}{2}g\left(t - \frac{1}{4}\right)^2$ ... (ii)

From eq. (ii),

$20 = \frac{1}{4}gt - \frac{1}{32}g$

$= 8t - 1 \quad [\because g = 32 \text{ ft/s}^2]$

$\therefore \quad t = \frac{21}{8}$

Then, from eq. (i), we get h = 110.25 ft.

139. (a) Let E be the event that a six occurs and A the event that the man reports that it is a '6'.

Therefore, $P(E) = \frac{1}{6}, P(E') = \frac{5}{6}$

Now, man speak the truth 3 times out of 4 times.

$\therefore$ Prob. (speak the truth and reports that it is 6) = $P(A/E) = \frac{3}{4}$ and $P(A/E') = \frac{1}{4}$

$\therefore$ From Baye's theorem, $P(E/A) = \frac{3}{8}$

140. (c) Total no. of cards = 52

Total no. of ace cards = 4

$\therefore$ P (drawing ace cards) = $\dfrac{4}{52} = \dfrac{1}{13} \equiv p$

and $q = 1 - p = 1 - \dfrac{1}{13} = \dfrac{12}{13}$

Here, we can apply Binomial distribution,

with $n = 2$, $p = \dfrac{1}{13}$, $q = \dfrac{12}{13}$

($n = 2$ as two cards are drawn)

We know, mean of Binomial distribution

$= np = 2 \times \dfrac{1}{13} = \dfrac{2}{13}$

141. (a) Let the first sphere be

$S_1 \equiv x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$, and second sphere be

$S_2 \equiv x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$,

By comparing the given equation of spheres with

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

We get centres of S_1 and S_2 as

$c_1 \equiv (-3, 4, 1)$ $\quad c_2 \equiv (5, -2, 1)$

So, mid point of line c_1c_2 (say)

$P \equiv \left(\dfrac{5-3}{2}, \dfrac{4-2}{2}, \dfrac{1+1}{2}\right) \Rightarrow P = (1, 1, 1)$

Now the plane $2ax - 3ay + 4az + 6 = 0$ passes throughout the point p, so point P (1, 1, 1) satisfies the equation of the plane.

$\therefore$ put $x = 1$, $y = 1$, $z = 1$ in equation of plane, we get $a = -2$

142. (b) **Note :** The projection of the join of the point $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$ on line AB whose direction cosines are ℓ, m, n is

$\ell(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

$\therefore$ In given question direction cosines of the line are $\dfrac{6}{7}, \dfrac{2}{7}, \dfrac{3}{7}$.

$\therefore$ projection of the line segment joining the points on the given line

$= \dfrac{6}{7}(2+1) + \dfrac{2}{7}(5-0) + \dfrac{3}{7}(1-3) = \dfrac{22}{7}$.

143. (b) Let the required vector be $a\hat{i} + b\hat{j} + c\hat{k}$.

Since $a\hat{i} + b\hat{j} + c\hat{k}, \hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ are coplanar. (given)

$\therefore$ Determinant made by these vectors is equal to zero.

$$\Rightarrow \begin{vmatrix} a & b & c \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

On solving determinant, we get $a - b + c = 0$

Also, given that required vector is parallel to the vector $2\vec{i} - 2\vec{j} - 4\vec{k}$. therefore, cross product of these two vectors is zero.

i.e., $(a\hat{i} + b\hat{j} + c\hat{k}) \times (2\hat{i} - 2\hat{j} - 4\hat{k}) = 0$

$$\text{i.e., } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 2 & -2 & -4 \end{vmatrix} = 0$$

$\Rightarrow -4b + 2c = 0,\ 4a + 2c = 0,\ 2a + 2b = 0$

$\Rightarrow \dfrac{c}{2} = \dfrac{b}{1}, \dfrac{c}{2} = \dfrac{a}{-1}, \dfrac{a}{-1} = \dfrac{b}{1}$

i.e., $\dfrac{a}{-1} = \dfrac{b}{1} = \dfrac{c}{2}$ or $\dfrac{a}{1} = \dfrac{b}{-1} = \dfrac{c}{-2}$

$\therefore$ Required vector is $\hat{i} - \hat{j} - 2\hat{k}$

144. (a) Given equation of lines are

$xy + 2x + 2y + 4 = 0$ ... (i)

and $x + y + 2 = 0$ ... (ii)

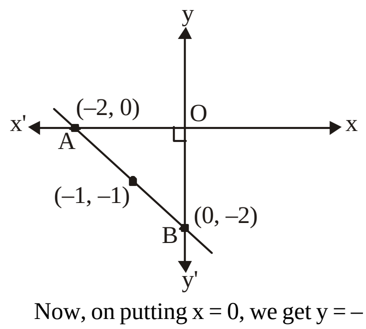
Equation (i) can be re-written as

$xy + 2(x + y + 2) = 0$

From equation (ii), we have

$xy + 2(0) = 0 \Rightarrow xy = 0$

$\Rightarrow x = y = 0$



Now, on putting $x = 0$, we get $y = -2$

and by putting $y = 0$, we get $x = -2$

$\therefore$ vertices of triangle are $(-2, 0), (0, 0), (0, -2)$.

Note : In a right angle triangle circumcentre is mid point of hypotenuse.
$\therefore$ Mid point of AB = (– 1, – 1)
Hence, (– 1, – 1) is the circumcentre.

145. (b) Given point p (a, b) lies on $3x + 2y = 13$
$\therefore$ (a, b) satisfies the equation
$3x + 2y = 13$
$\therefore$ $3a + 2b = 13$ ... (i)
and point Q (b, a) lies on $4x - y = 5$
$\therefore$ (b, a) satisfies the equation $4x - y = 5$.
$4b - a = 5$ ... (ii)
On solving equation (i) and (ii), we get
$a = 3, b = 2$
$\therefore$ p (a, b) = (3, 2) and Q (b, a) = (2, 3)
Now, equation of PQ is found by using the equation of two point form $x + y = 5$.

146. (c) **Note :** The equation of the bisectors of the acute angle between the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = -\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Therefore, the equation of bisector of acute angle formed between the lines $4x - 3y + 7 = 0$ and $3x - 4y + 14 = 0$, is

$$\frac{4x - 3y + 7}{\sqrt{16 + 9}} = -\frac{3x - 4y + 14}{\sqrt{16 + 9}}$$

$\Rightarrow x - y + 3 = 0$

147. (b) **Note :** The line $y = mx + c$ will touch the circle $x^2 + y^2 = a^2$ if $c^2 = a^2 (1 + m^2)$
Given equation of line is $y \cos\alpha = x \sin\alpha + a \cos\alpha$
$\Rightarrow y = x \tan\alpha + a$
On comparing with $y = mx + c$, we get
$m = \tan\alpha$ and $c = a$
$\therefore$ From the note, we have
$a^2 = a^2 [1 + \tan^2\alpha]$
$\Rightarrow \sec^2\alpha = 1 \Rightarrow \cos^2\alpha = 1$

148. (b) Given parametric equations of curve are $x = t^2 + 1$ and $y = 2t$
Eliminating t from $x = t^2 + 1$, $y = 2t$, we have

$y = 2t \Rightarrow \frac{y}{2} = t$

Put $t = \frac{y}{2}$ in the curve $x = t^2 + 1$

we have $x = \left(\frac{y}{2}\right)^2 + 1$

$y^2 = 4x - 4$
Similarly other parametric equations of curve are $x = 2s$ and $y = \frac{2}{s}$

Eliminating s from $x = 2s$, $y = \frac{2}{s}$, we get
$xy = 4$
Thus, we have $y^2 = 4x - 4$ and $xy = 4$
On solving these curves, we get point of intersection is (2, 2)

149. (d) Standard form of the ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Focus = (ae, 0), eccentricity = e

Directrix : $x = \frac{a}{e}$ and $b^2 = a^2 (1 - e^2)$

Given, $2ae = 8$ and $\frac{2a}{e} = 18 \Rightarrow e = \frac{2}{3}$

$\therefore a = 6$ and $b = 2\sqrt{5}$
Therefore the required equation of ellipse is $\frac{x^2}{36} + \frac{y^2}{20} = 1 \Rightarrow 5x^2 + 9y^2 = 180$

150. (a) Given : $\cos\theta = \frac{8}{17}$ and $0 < \theta < \frac{\pi}{2}$

As we know, $\sin\theta = \sqrt{1 - \cos^2\theta}$

$\therefore \sin\theta = \sqrt{1 - \frac{8^2}{17^2}} = \frac{15}{17}$

Consider the given expression, viz
$\cos(30°+\theta) + \cos(45°-\theta) + \cos(120°-\theta)$
$= \cos 30° . \cos\theta - \sin 30° \sin\theta + \cos 45° \cos\theta + \sin 45° \sin\theta + \cos 120° \cos\theta + \sin 120° \sin\theta$
(By using $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and $\cos(A - B) = \cos A \cos B + \sin A \sin B$)

$$= \cos\theta\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right) - \sin\theta\left(\frac{1}{2} - \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\right)$$

$$= \frac{8}{17}\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right) + \frac{15}{17}\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

$$= \frac{23}{17}\left(\frac{\sqrt{3} - 1}{2} + \frac{1}{\sqrt{2}}\right)$$

151. (c) Consider the Expansion $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$

On comparing with $(x + a)^n$, we get

$x = 1,\ a = \frac{1}{\sqrt{3}}, n = 20.$

Now, Let T_r and T_{r+1} be rth and (r + 1)th terms of given expansion

$\Rightarrow\ T_{r+1} = {}^{20}C_r\ (1)^{20-r}.\left(\frac{1}{\sqrt{3}}\right)^r.\sqrt{3}$ and

$T_r = {}^{20}C_{r-1}\ (1)^{20-r+1}.\left(\frac{1}{\sqrt{3}}\right)^{r-1}.\sqrt{3}$

Now, $\frac{T_{r+1}}{T_r} = \frac{\sqrt{3}.\left(\frac{1}{\sqrt{3}}\right)^r.{}^{20}C_r}{\sqrt{3}.\left(\frac{1}{\sqrt{3}}\right)^{r-1}{}^{20}C_{r-1}}$

$= \frac{1}{\sqrt{3}} \times \frac{20 - r + 1}{r}$

As $T_{r+1} \ge T_r \Rightarrow \frac{T_{r+1}}{T_r} \ge 1$

$\Rightarrow\ r \le 7.69\ \Rightarrow\ r = 7$

Thus, T_8 is the greatest term.

i.e. $T_8 = \frac{25840}{9}$

152. (c) **Note :** If $Z = x + iy$ be a complex number, then

amp (z) = $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

Given, $\sin\frac{\pi}{5} + i(1 - \cos\frac{\pi}{5})$

$= \sin\frac{\pi}{5} + i\left[2\sin^2\frac{\pi}{10}\right]$

$= 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + i\left[2\sin^2\frac{\pi}{10}\right]$

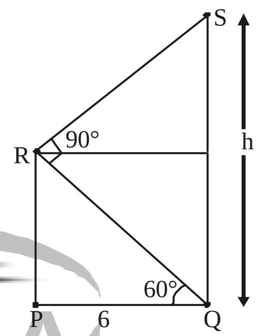
(By using $\sin 2\theta = 2\sin\theta\cos\theta$)

$\therefore$ amp$\left[\sin\frac{\pi}{5} + i(1 - \cos\frac{\pi}{5})\right]$

$= \tan\theta = \tan\frac{\pi}{10}\ \Rightarrow\ \theta = \frac{\pi}{10}$

153. (a) Let P be the second and QS be the first house.

Let R be the position of window. Let 'h' be the height of first house.



i.e. QS = h m.

Given : PQ = 6m.

and $\angle$ RQP = 60°, $\angle$ SRQ = 90°

Now, in ΔRPQ,

$\frac{RP}{PQ} = \tan 60° \Rightarrow \frac{RP}{6} = \sqrt{3} \Rightarrow RP = 6\sqrt{3}$m

$\therefore$ By using Pythagorus theorem in Δ PQR, we have

$(RQ)^2 = (6\sqrt{3})^2 + (6)^2 = 144 \Rightarrow RQ = 12$m

Now, in ΔRQS, $\sin 60° = \frac{12}{h} \Rightarrow h = 8\sqrt{3}$m.

Hence, height of the first house = $8\sqrt{3}$m

154. (b) Total Given nos. are 200.

$\therefore$ Total no. of two factors products $= {}^{200}C_2$

Now, the number of numbers from 1 to 200 which are multiples of 5 is 40.

$\therefore$ nos. which are not multiples of 5 is 160.

$\therefore$ Total no. of two factors product, which are not multiple of 5 is ${}^{160}C_2$.

$\therefore$ Required nos. of factor

$= {}^{200}C_2 - {}^{160}C_2 = 19900 - 12720 = 7180.$

155. (b) Let $\vec{a}$ be any vector

Such that $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Let $\hat{i}$ be a unit vector in the direction of $\vec{b}$ and $\hat{j}$ in the direction of $\vec{c}$.

$\therefore\ \vec{b} = \vec{i}$ and $\vec{c} = \vec{j}$

$\Rightarrow |\vec{b}\times\vec{c}| = \sin\alpha \; (\because |b| = 1, |c| = 1)$

Now, $\vec{a}.\vec{b} = \vec{a}.\hat{i} = a_1$ …(1)

and $\vec{a}.\vec{c} = \vec{a}.\hat{j} = a_2$ …(2)

and $\vec{a}.\dfrac{\vec{b}\times\vec{c}}{|\vec{b}\times\vec{c}|} = \vec{a}.\hat{k} = a_3 \; (\because \vec{i}\times\vec{j} = \vec{k})$ …(3)

Consider

$$(\vec{a}.\vec{b})\vec{b} + (\vec{a}.\vec{c})\vec{c} + \frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}.(\vec{b}\times\vec{c})$$

$= a_1\vec{b} + a_2\vec{c} + a_3(\vec{b}\times\vec{c})$

from (1), (2) and (3)

$= a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \vec{a}$

156. (d) Let S denote the sum of the series upto ∞.

$\therefore S = 1 + 3x + 6x^2 + 10x^3 + \ldots..\infty$ (i)

Multiplied both side by x and subtract from equation (i), we get

$S(1-x) = 1 + 2x + 3x^2 + 4x^3 + \ldots..\infty$

$x(1-x)S = x + 2x^2 + 3x^3 + \ldots..\infty$

Again subtracting

$S[(1-x) - x(1-x)] = (1 + x + x^2 + x^3 + \ldots.\infty)$

$\Rightarrow S[(1-x)(1-x)] = \dfrac{1}{1-x} \Rightarrow S = \dfrac{1}{(1-x)^3}$

157. (c) Given expression is

$$\left|\frac{1}{2}(z_1+z_2)+\sqrt{z_1z_2}\right| + \left|\frac{1}{2}(z_1+z_2)-\sqrt{z_1z_2}\right|$$

$$= \frac{1}{2}\left|z_1+z_2+2\sqrt{z_1z_2}\right| + \frac{1}{2}\left|z_1+z_2-2\sqrt{z_1z_2}\right|$$

$$= \frac{1}{2}|(\sqrt{z_1}+\sqrt{z_2})^2| + \frac{1}{2}|(\sqrt{z_1}-\sqrt{z_2})^2|$$

$$= \frac{1}{2}|\sqrt{z_1}+\sqrt{z_2}|^2 + \frac{1}{2}|\sqrt{z_1}-\sqrt{z_2}|^2$$

$\{\because |z^2| = |z|^2\}$

$$= \frac{1}{2}2\left[|\sqrt{z_1}|^2 + |\sqrt{z_2}|^2\right]$$

$[\because |z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2 + |z_2|^2)]$

$= |z_1| + |z_2|$

158. (b) Consider

$\lim\limits_{x\to0} x\log_e(\sin x) = \lim\limits_{x\to0} x\log_e(\sin x)^x$

$= \log_e[\lim\limits_{x\to0}(\sin x)^x] \; (\because \lim\limits_{x\to0}(\sin x)^x > 0)$

Add and subtract 1, we get

$= \log_e[\lim\limits_{x\to0}(1+\sin x-1)^x]$

Now, multiply and divide by $\sin x - 1$ in the power.

$$= \log_e\left[\lim_{x\to0}(1+\sin x-1)^{\frac{x(\sin x-1)}{(\sin x-1)}}\right]$$

$$= \log_e\left[e^{\lim\limits_{x\to0} x(\sin x-1)}\right] = \log_e 1$$

159. (b) $f(x) = \begin{cases}\left(x^2 + e^{\frac{1}{2-x}}\right)^{-1}, & x \neq 2\\ k, & x = 2\end{cases}$

$\Rightarrow f(x) = \left[x^2 + \dfrac{1}{e^{1/2-x}}\right]^{-1}$ and $f(2) = k$

Since, f (x) is continuous from right at x = 2, therefore $\lim\limits_{x\to2^+} f(x) = f(2) = k$

Put x = 2 + h in f (x) and when $x \to 2$, then $h \to 0$

$\Rightarrow k = \lim\limits_{h\to0} f(2+h)$

$\Rightarrow k = \lim\limits_{h\to0}\left[(2+h)^2 + e^{1/2-(2+h)}\right]^{-1}$

$\Rightarrow k = \dfrac{1}{4}$

160. (a) Let $I = \displaystyle\int\frac{dx}{\sin(x-a)\sin(x-b)}$

Multiplied and divided by sin (a – b)

$$\Rightarrow I = \frac{1}{\sin(a-b)}\int\frac{\sin(a-b)dx}{\sin(x-a)\sin(x-b)}$$

Now, add and subtract x.

$$\Rightarrow I = \frac{1}{\sin(a-b)}\int \frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)}dx$$

Using sin(A–B) = sin A cos B – cos A sin B we have

$$= \frac{1}{\sin(a-b)}$$

$$\int \frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}dx$$

$$= \frac{1}{\sin(a-b)}\left[\int \cot(x-a)dx - \int \cot(x-b)dx\right]$$

$$= \frac{1}{\sin(a-b)}\,[\log\sin(x-a) - \log\sin(x-b)] + c$$

$$= \frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + c$$

161. (a) Let $I = \int_0^a \frac{x^4 dx}{(a^2+x^2)^4}$

Put $x = a\tan\theta \Rightarrow dx = a\sec^2\theta\, d\theta$

$\therefore$ we have

$$I = \frac{1}{a^3}\int_0^{\pi/4} \sin^4\theta.\cos^2\theta d\theta$$

$$= \frac{1}{a^3}\int_0^{\pi/4} \sin^4\theta(1-\sin^2\theta)d\theta$$

$$= \frac{1}{a^3}\int_0^{\pi/4} (\sin^4\theta - \sin^6\theta)\, d\theta$$

$$= \frac{1}{a^3}\int_0^{\pi/4}\left[\frac{(1-\cos 2\theta)^2}{4} - \frac{(1-\cos 2\theta)^3}{8}\right]d\theta$$

$$= \frac{1}{a^3}\int_0^{\pi/4} \frac{(1-\cos 2\theta)^2(2-1+\cos 2\theta)}{8}d\theta$$

$$= \frac{1}{8a^3}\int_0^{\pi/4} (1-\cos 2\theta - \cos^2 2\theta + \cos^3 2\theta)d\theta$$

$$= \frac{1}{8a^3}\int_0^{\pi/4}\left[1-\cos 2\theta - \left(\frac{\cos 4\theta + 1}{2}\right) + \frac{3\cos 2\theta}{4} + \frac{\cos 6\theta}{4}\right]d\theta$$

$$= \frac{1}{8a^3}\int_0^{\pi/4}\frac{1}{2} - \frac{\cos 2\theta}{4} - \frac{\cos 4\theta}{2} + \frac{\cos 6\theta}{4}\, d\theta$$

(By using $\cos 2\theta = 2\cos^2\theta - 1$,
$\cos 3\theta = -3\cos\theta + 4\cos^3\theta$)

$$= \frac{1}{32a^3}\int_0^{\pi/4} (2 - \cos 2\theta - 2\cos 4\theta + \cos 6\theta)d\theta$$

$$= \frac{1}{32a^3}\left[2\theta - \frac{\sin 2\theta}{2} - \frac{\sin 4\theta}{2} + \frac{\sin 6\theta}{6}\right]_0^{\pi/4}$$

$$= \frac{1}{32a^3}\left[\frac{\pi}{2} - \frac{1}{2} + \frac{1}{6}\sin\frac{3\pi}{2}\right] = \frac{1}{16a^3}\left(\frac{\pi}{4} - \frac{1}{3}\right)$$

162. (a) The given differential equation is

$$(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0$$

$$\Rightarrow x^2(1-y)\,dy = -(1+x)\,y^2 dx$$

Separating the variables

$$\frac{1-y}{y^2}dy = -\left(\frac{1+x}{x^2}\right)dx$$

On integrating, we get

$$-\frac{1}{y} - \log y = -\left(-\frac{1}{x} + \log x\right) + c$$

$$\Rightarrow \log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$$

163. (b) Given equation is $\frac{dy}{dx} = \frac{x+y}{x-y}$ ... (i)

Since, it is a homogeneous equation,
$\therefore$ put y = vx and by differentiating, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Put value of $\frac{dy}{dx}$ in equation (i), we get

$$x\frac{dv}{dx} = \frac{1+v^2}{1-v}$$

$$\Rightarrow \frac{1}{x}dx = \left(\frac{1}{1+v^2} - \frac{v}{1+v^2}\right)dv$$

On integrating both side, we get

$$\Rightarrow \quad \log_e x = \tan^{-1} v - \frac{1}{2}\log_e(1+v^2) - \log_e c$$

Put value of v

$$\log_e x = \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log_e\left[1+\left(\frac{y}{x}\right)^2\right] - \log_e c$$

$$\Rightarrow \; c\,(x^2+y^2)^{1/2} = e^{\tan^{-1}(y/x)}$$

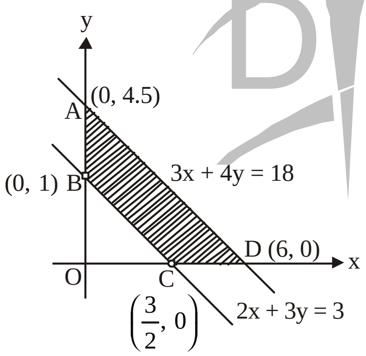
164. (d) Given, linear constraints are

$3x + 4y \le 18$ $2x + 3y \ge 3$ and $x \ge 0, y \ge 0$.

Now, by changing them into equality and by giving some values of x and y, we find co-ordinates to draw the graph of these constraints.

Shaded portion shows feasible region and vertices of the feasible region are

A (0, 4.5), B (0, 1), $C\left(\frac{3}{2}, 0\right)$ and D (6, 0).



165. (b) Given, LP problem is

Min $z = 2x + y$

Subject to $5x + 10y \le 50$

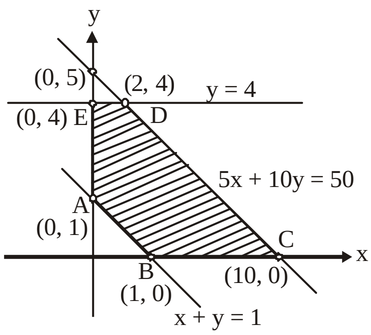
$x + y \ge 1$

$y \le 4$

$x, y \ge 0$

Now, we draw the graph of these constraints by converting them in equality and draw the feasible region.

Shaded portion shows feasible region ABCDEA and vertices of the feasible region are A (0, 1), B (1, 0), C (10, 0), D (2, 4), E (0, 4).



Now, objective function $z = 2x + y$.

At A (0, 1), z = 1

At B (1, 0), z = 2

At C (10, 0), z = 20

At D (2, 4), z = 8

At E (0, 4), z = 4

Hence, minimum value of z at (0, 1) is 1.

166. (b) Given planes are $\vec{r}.\vec{a} = \lambda$ and $\vec{r}.\vec{b} = \mu$

$$\Rightarrow \quad \vec{r}.\vec{a} + \lambda = 0 \text{ and } \vec{r}.\vec{b} - \mu = 0$$

The equation of the plane through the line of intersection of the given plane can be written as $(\vec{r}.\vec{a} - \lambda) + k(\vec{r}.\vec{b} - \mu) = 0$ where 'k' is a scalar.

$$\Rightarrow \quad \vec{r}.(\vec{a} + k\vec{b}) = \lambda + k\mu \qquad \ldots \text{(i)}$$

Also this passes through the origin, therefore, we have $k = -\frac{\lambda}{\mu}$

Put the value of k in eq. (i), we get

$$\vec{r}.(\lambda\vec{b} - \mu\vec{a}) = 0$$

which is the required equation of plane.

167. (c) **Note :** Angle between two lines

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where the direction cosines of two lines be proportional to a_1, b_1, c_1 and a_2, b_2, c_2.

In the given question,

$a_1 = 1, b_1 = 2, c_1 = 3$

$a_2 = 3, b_2 = -2, c_2 = 1$

$$\therefore \cos\theta = \frac{1\times 3 + 2\times(-2) + 3\times 1}{\sqrt{1^2+2^2+3^2}\sqrt{3^2+(-2)^2+1^2}}$$

$$= \frac{2}{\sqrt{14}\sqrt{14}}$$

$$\therefore\ \theta = \cos^{-1}\left(\frac{1}{7}\right)$$

168. (c) Given, $f(x) = 2x^3 - 21x^2 + 36x - 30$
Differentiate both side, w.r.t 'x'.
$\Rightarrow\ f'(x) = 6x^2 - 42x + 36$
For maxima or minima, put $f'(x) = 0$
$\Rightarrow\ x = 6, 1$
Now, $f''(x) = 12x - 42$
and $f''(1) = -30$ and $f''(6) = 30$
$f''(1) = -30$ is negative, therefore $f(x)$ has maxima at $x = 1$ and $f''(6) = 30$ is positive, therefore $f(x)$ has minima at $x = 6$.

169. (b) As we know eccentricity of the hyperbola is always greater than 1.
$\therefore$ From the options, we have one of the eccentricity is $2\sqrt{\frac{1}{9}} = \frac{2}{3}$ which is less than 1.
$\therefore$ Eccentricity of hyperbola can never be equal to $\frac{2}{3}$.

170. (c) **Note :** The straight line $y = mx + c$ touches the hyperbola if $c = \pm\sqrt{a^2m^2 - b^2}$
Here, given line is $y = 2x + \lambda$
$\therefore\quad m = 2, c = \lambda$
Given, hyperbola is $36x^2 - 25y^2 = 3600$

$$\Rightarrow\ \frac{x^2}{100} - \frac{y^2}{\frac{3600}{25}} = 1$$

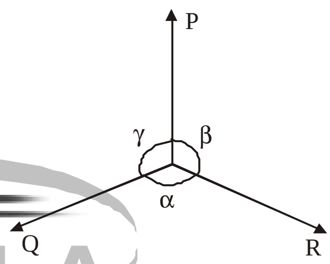
$\therefore\ a^2 = 100, b^2 = 144$
Put all the values, in the formula, we get
$\lambda = \pm\sqrt{100(4) - 144} = \pm 16.$

171. (a) Consider $\sim(p \vee q) \vee (\sim p \wedge q)$
$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$
$\equiv \sim p \wedge (\sim q \vee q) \equiv \sim p$

172. (d) From the given circuit, We have Boolean polynomial is $(\sim p \wedge q) \vee (p \wedge \sim q)$.

173. (c) Given observations are
$-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5.$
Also given, standard deviation of above observation is $\sqrt{10}$.
The new observations are obtained by adding 20 to each. Thus, by adding some constant in observations, the standard deviation of new observation does not change. Therefore, σ does not change.

174. (d) **Note : Lami's theorem**

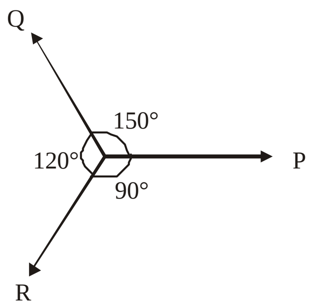


If the forces P, Q, R act at a point and α, β, γ be the angles between Q, R; R, P and P, Q and forces are in equilibrium then

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Here angle are 120°, 90° and 150°
By Lami's theorem

$$\frac{P}{\sin 120^\circ} = \frac{Q}{\sin 90^\circ} = \frac{R}{\sin 150^\circ}$$



$$\Rightarrow \frac{P}{\sqrt{3}/2} = \frac{Q}{1} = \frac{R}{1/2} \Rightarrow P : Q : R = \sqrt{3} : 2 : 1$$

175. (c) Total no. of persons = 5

$\therefore$ Total number of ways = 5!

when A and E stands together, then we treat A and E as one person.

$\therefore$ Total no. of persons = 4

$\therefore$ Favourable number of ways = 2 .4!

$\therefore$ P (A and E always together) $= \frac{2.4!}{5!} = \frac{2}{5}$

176. (a) Let $f : N \to N$ defined as

$f(x) = x^2 + x + 1 \;\forall\; x \in N$

Note : f (x) is said to be one-one if $f(a) = f(b)$

$\Rightarrow a = b \;\forall\; a, b \in N$ and f (x) is said to be onto if we have $y \in$ co-domain then $\exists\, x \in$ domain such that $f(x) = y$

Let $x, y \in N$ such that $f(x) = f(y)$

$\Rightarrow \quad x^2 + x + 1 = y^2 + y + 1$

$\Rightarrow \quad (x - y)(x + y) + (x - y) = 0$

$\Rightarrow \quad (x - y)(x + y + 1) = 0$

$\Rightarrow \quad x = y$ or $x = (-y - 1) \notin N$

$\therefore \quad$ f is one-one

Again, since for each $y \in N$, there exist $x \in N$

$\therefore \quad$ f is onto.

177. (c) **Note 1:** If a, b, c are three numbers in H.P. then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Note 2 : If a, b, c are three numbers in A.P. then $b - a = c - b$

Note 3 : In any ΔABC, we have

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$$

and $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

We have $\frac{1}{\sin^2\frac{A}{2}}, \frac{1}{\sin^2\frac{B}{2}}, \frac{1}{\sin^2\frac{C}{2}}$ are in AP. [from Note (1)]

$$\Rightarrow \frac{1}{\sin^2\frac{C}{2}} - \frac{1}{\sin^2\frac{B}{2}} = \frac{1}{\sin^2\frac{B}{2}} - \frac{1}{\sin^2\frac{A}{2}}$$

[from Note (2)]

$$\Rightarrow \frac{ab}{(s-a)(s-b)} - \frac{ac}{(s-a)(s-c)}$$

$$= \frac{ac}{(s-a)(s-c)} - \frac{bc}{(s-b)(s-c)}$$

[from note (3)]

$\Rightarrow abs - abc - acs + abc$

$= acs - abc - bcs + abc$

$\Rightarrow ab + bc = 2ac \Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$

$\Rightarrow$ a, b, c are in HP.

178. (b) **Note :** In any Δ ABC, we have

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

and $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$

Consider $\dfrac{\tan\frac{A}{2} - \tan\frac{B}{2}}{\tan\frac{A}{2} + \tan\frac{B}{2}}$

$$= \frac{\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} - \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}}{\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} + \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}}$$

$$= \sqrt{\frac{s(s-c)[(s-b)^2 - (s-a)^2]}{s(s-c)[(s-b)^2 + (s-a)^2]}}$$

$$= \frac{\sqrt{(s-b)^2} - \sqrt{(s-a)^2}}{\sqrt{(s-b)^2} + \sqrt{(s-a)^2}}$$

$= \frac{a-b}{c} \qquad (\because 2s = a + b + c)$

179. (c) Given equation is $y^2 - x^2 + 2x - 1 = 0$

Second degree equation is given as $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

By comparing original equation with given equation we get, $a = -1$, $h = 0$, $b = 1$, $f = 0$, $g = 1$, $c = -1$

We know that Δ is given as :

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

By putting the values of a, b, c, f, g and h in Δ, we get $\Delta = 0$

Since, $\Delta = 0$ therefore, the given equation represents two straight lines.

180. (b) Given expression is

$$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$$

Multiplying the numerator and denominator by ω and ω^2 respectively of I and II expression.

$$=\frac{\omega(a+b\omega+c\omega^2)}{\omega(b+c\omega+a\omega^2)}+\frac{\omega^2(a+b\omega+c\omega^2)}{\omega^2(c+a\omega+b\omega^2)}$$

$$=\frac{\omega(a+b\omega+c\omega^2)+\omega^2(a+b\omega+c\omega^2)}{(a+b\omega+c\omega^2)}$$

$(\because \omega^3 = 1)$

$$=\frac{(a+b\omega+c\omega^2)(\omega+\omega^2)}{(a+b\omega+c\omega^2)}$$

$= \omega + \omega^2$

$= -1$ $[\because 1 + \omega + \omega^2 = 0]$